UNIVERSITÉ DU QUÉBEC EN OUTAOUAIS

AN AGENT-BASED MODEL OF THE CANADIAN HOUSING MARKET

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MÉMOIRE DE RECHERCHE PRÉSENTÉ AU DÉPARTEMENT DES SCIENCES ADMINISTRATIVES COMME EXIGENCE PARTIELLE DE LA MAÎTRISE EN ÉCONOMIE FINANCIÈRE

MARS 2023

RÉSUMÉ

Des travaux antérieurs montrent qu'un modèle basé sur les agents, appelé en anglais « *Agent-Based Model* (ABM) » peut reproduire les principales caractéristiques du marché immobilier anglais et étudier les effets que les chocs économiques ont sur ce marché. Nous étendons ce modèle et créons un ABM pour le marché immobilier canadien en utilisant quarante-huit ans de données empiriques sur l'économie canadienne comme variables d'entrée (le taux d'intérêt hypothécaire, le revenu disponible des ménages et le taux d'inflation). Nous constatons que le modèle ABM étendu produit des estimations convergentes successives des prix des propriétés résidentielles en équilibre à long terme avec les données réelles observées sur les prix des maisons au Canada. Nos résultats suggèrent que les taux d'intérêt hypothécaires sont négativement corrélés avec les prix des maisons, tandis que les revenus des ménages et l'inflation sont fortement corrélés positivement avec les prix des maisons. En outre, les ABMs ont des applications pratiques pour l'analyse de scénarios et pour éclairer les politiques macroprudentielles, par exemple en limitant l'effet de levier ou en ajoutant des tests de résistance aux taux hypothécaires.

Mots clés : marché immobilier canadien, modèle basé sur les agents, prix des maisons, taux d'intérêt hypothécaire, taux d'inflation, revenu disponible des ménages, politiques macroprudentielles, NetLogo

ABSTRACT

Previous work shows that an agent-based model (ABM) can reproduce the main characteristics of the English housing market and study the effects that economic shocks have on this market. We extend this model and create an ABM for the Canadian housing market using forty-eight years of empirical data on the Canadian economy as input variables (mortgage interest rate, household disposable income and inflation rate). We find that the extended ABM model produces successive convergent estimates of residential property prices in long-run equilibrium with the actual observed property price data in Canada. Our results suggest that mortgage interest rates are negatively correlated with house prices, while household incomes and inflation are strongly positively correlated with house prices. Furthermore, ABMs have practical applications for scenario analysis and to inform macroprudential policies, for example limiting leverage or adding mortgage rate stress tests.

Keywords: Canadian housing market, Agent-based model, house prices, mortgage interest rate, inflation rate, household disposable income, macroprudential policies, NetLogo

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ACRONYMS AND ABBREVIATIONS

ABM	Agent-Based Model
BoC	Bank of Canada
BIS	Bank for International Settlements
CMB	Canada Mortgage Bond
CMHC	Canada Mortgage and Housing Corporation
COVID-19	Coronavirus Disease
CPI	Consumer Price Index
CV	Coefficient of Variation
CSV	Comma-Separated Values
DC	District of Columbia, United States of America
DSGE	Dynamic Stochastic General Equilibrium
DTI	Debt to Income Ratio
FRED	Economic Data from the Federal Reserve Bank of St. Louis
GDS	Gross Debt Service Ratio
HoMES	Housing Market Evolutionary System
HPI	House Price Index
ICR	Interest Cover Ratio
ILUTE	Integrated Land Use, Transportation, Environment
IMPP	Insured Mortgage Purchase Program

LTI	Loan to Income Ratio
LTV	Loan to Value Ratio
MBS	Mortgage-Backed Securities
NHA	National Housing Act
NHS	National Housing Strategy
NOS	National Occupancy Standard
OECD	Organisation for Economic Co-operation and Development
OSFI	Office of the Superintendent of Financial Institutions
Q1	First quarter of the year (January, February and March)
Q2	Second quarter of the year (April, May and June)
Q3	Third quarter of the year (July, August and September)
Q4	Fourth quarter of the year (October, November and December)
RW	Real-World
UK	United Kingdom
US	United States of America
WTP	Willingness to Pay

REMERCIEMENTS

Je tiens à remercier très sincèrement mon directeur de recherche, M. David Tessier, pour la direction, ses précieux conseils et son mentorat au-delà de ce mémoire. Je tiens également à remercier Mme Céline Gauthier, que je considère comme un modèle professionnel et qui m'a encouragée à poursuivre une maîtrise en économie financière, alors que ma formation précédente était dans le domaine de l'informatique.

Je suis très reconnaissante à tous mes professeurs et professeures de l'Université du Québec en Outaouais (UQO) pour tous les enseignements de chaque cours, y compris les cours préalables suivis avant de commencer le programme. Je remercie également les membres du jury pour l'évaluation de ce mémoire, que représente l'accomplissement d'un projet conjoint lancé il y a quelques années : une maîtrise dans le domaine de l'économie financière.

Je remercie mon amour Luciano Schwalbe, pour avoir embarqué avec moi dans ce projet. Je le remercie de tout mon cœur pour son soutien inconditionnel, l'encouragement dans les moments les plus critiques, et pour toujours vouloir continuer à apprendre ensemble. Je remercie également nos familles et amis pour tous les encouragements et aussi pour la patience de nous voir souvent continuer à étudier pendant les fins de semaine.

Finalement, j'aimerais exprimer ma gratitude à tout le personnel de l'UQO pour offrir l'opportunité de suivre un programme d'excellente qualité en économie financière, une occasion extraordinaire de développement professionnel et personnel.

INTRODUCTION

In many countries, the housing market represents an important sector of the economy. This is not different in Canada, where the real estate sector represents an increasingly growing portion of the national wealth[1]. The Canadian housing sector is not only a major source of direct and indirect jobs, but also considered by many Canadians as the most important investment they hold, as houses become a source of (perceived) wealth, specially during periods of boom markets. After the 2008 housing crisis in the United States, this sector has gained additional attention, as evidence indicates that boom-busts in the housing market may precede larger economic crises when they are financed through credit and involve leverage[2,3].

The proper identification of market trends and the development of macroprudential policies can be applied an effective response to mitigate specific sources of risk. It would be valuable to have the possibility of simulating the effects of such measures in a model that closely represents the real-life economy to prove effectiveness of macroprudential measures in development and identify possibilities of circumvention, as an example.

The development of economic models allows the representation of complex processes and economy sectors in a simplified way, so problems and behaviours can be analysed. Different models are suitable for specific purposes. As an example, the classic Dynamic Stochastic General Equilibrium (DSGE) models are built based on equilibrium and assume agents are rational and homogeneous, looking to maximize their utility. Most macroeconomic models, including housing market models, are built using DSGE models. However, the development of computational technology and the increasing integration of psychology to the economy field (behavioural economics) allowed for the emergence of alternative economic

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1 See Statistics Canada (2018).
2 See Crowe, Giovanni, Igan, and Rabanal (2013).
3 See Ge (2017).

models that complement DSGE models. One of the computational models that have been progressively developed and applied for modelling the housing markets are the agent-based models (ABMs).

ABMs are computational models that allow the representation of an economy with heterogeneous adaptive agents that interact autonomously with each other, while also reacting to the environment in which they are inserted. These successive interactions generate emergent collective behaviours. Based on endogenous properties and on agents that are not constrained to following a rational behaviour, these models have been proving themselves very well suited for simulation of economies in a smaller scale.

Agent-based models of the housing market have been applied in different countries, including England, the United States and Denmark. In Canada, an ABM was developed for the Toronto-Hamilton area[4], but not for the whole Canadian housing market. Models of the housing market are built allowing the representation of households' properties and their actions depending on environment variables and some observed (or surveyed) behaviours. Consequently, it is necessary to calibrate the model with data that properly represents the economy and the period being studied. The objective of this work is to propose and develop an agent-based model of the Canadian housing market, calibrated with data from the Canadian economy, the Canadian housing sector and reflecting Canadian behaviours. Although the shock caused by the COVID-19 pandemic has affected the Canadian housing market in its entirety, Canada has traditionally presented a heterogeneous housing market (regional housing prices being weakly correlated). It is recommended to consider the specific regional data and factors for understanding the Canadian housing market cycles. These requirements can be supported in an ABM, due to the heterogeneity and multidimensional perspective these models can offer.

There are endless possibilities for ABMs, which would theoretically allow the representation of an economy where agents could represent each individual household in the defined geographic location where they live. Because a model with such level of detail would require massive computational power to run, in addition to extensive calibration, ABMs are

[4] See Rosenfield, Chingcuanco, and Miller (2013).

usually created as simplified versions of the real-life, which can then be iteratively incremented to the needed level of complexity. ABMs also have some challenges, since these models are difficult to develop and review, and they may become overparametrized[5]. For these reasons, these models are not usually very suitable for producing market forecasts.

Since ABMs are appropriate for simulations and the observation of emergent collective behaviours, they are a very good tool for the study of the effects of macroprudential regulation and economic shocks. In Canada, a relevant application of an ABM of the housing market would be the simulation of the market trends with and without some macroprudential limits (one relatively recent limit being the stress tests with higher interest rate scenarios introduced in 2016 to insured mortgages and extended in 2018 to non-insured mortgages[6,7]). Another possibility would be the simulation of the effects of the measures intended on addressing housing affordability across Canada[8,9,10]. This work aims for the development of a simplified model of the Canadian housing market, not focusing on a specific province or region. The objective is simulating the effects of macroprudential regulation while also identifying how some economic shocks would affect agents' emergent aggregate behaviour in different scenarios.

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5 See Gräbner (2016).
6 See OSFI (2017).
7 See Siddall (2019).
8 See Government of Canada (2019).
9 See Government of Canada (2021).
10 See Government of Canada (2022).

CHAPTER I

LITERATURE REVIEW

Agent-based models (ABMs) are a relatively recent approach for modelling complex and dynamic systems using autonomous interacting agents. These bottom-up models allow the study of emergent collective behaviours originated through successive heterogeneous agents' adaptations and interactions. ABMs are employed in different scientific fields, particularly when adaptiveness and emergence of agents' self-organization are important considerations [Macal and North (2010)]. While some ground-breaking studies on ABMs applied to housing markets have been completed, there is an opportunity of further development in this field, particularly in Canada. A review of the available literature will be presented in the next sections, starting with a review of ABMs in Economics, followed by an examination of ABMs applied to the housing market and concluding with an analysis of the Canadian housing market.

1.1 Agent-Based Models in Economics

Agent-based models are inherently complex adaptive systems. They are composed of heterogeneous autonomous agents that interact with each other and their environment. In these models, differently from DSGE models, general equilibrium is not an assumption. While classic DSGE models have a top-down construction, ABMs propose a bottom-up emergence of equilibrium from the system dynamics, through repeated local interactions of autonomous agents. The study of non-linear dynamics (or complex systems) and the developments of artificial intelligence are in the origins of ABMs. Holland and Miller (1991) developed one of the early works on Artificial Adaptive Agents (AAA's) in economic theory and found that they are a good complement for classical modeling theory, allowing the exploration of system dynamics with control of conditions, while checking unfolding behaviours for plausibility.

Economics ABMs consist of autonomous agents (usually representing individuals, households, firms or a group of them), relationships (the local network that connects the agents) and an underlying environment. According to Gräbner (2016), these models help understanding how individuals' actions lead to patterns and behaviours, and what dynamics result from the interactions of this societal system.

ABMs allow the development of heterogeneous systems (one of the key characteristics of ABMs). In the real world, societies and markets are composed of diverse individuals. As Thaler (2015) appropriately describes, human beings do not behave like "*Econs*" (a reference to the term "*homo economicus*"). While a society of "*Econs*" would probably develop homogeneous behaviours based on unbiased and complex rules to maximize their utility, typical human beings are less rational and much less sophisticated when making decisions. When contemplating human beings' decisions, a complex set of heterogeneous behaviours are present: they can be biased, and they may be short-sighted and time inconsistent.

As humans will also learn from past experiences and interactions, the adaptiveness of agents creates a better representation of the human behaviour in a model. To essentially represent the adaptiveness of human behaviour from the systems under investigation, agent-based computational models employ adaptive algorithms, which allow an artificial social system with autonomous and interacting agents to be brought to life. Different types of adaptive algorithms can be employed, including genetic algorithms, classifier systems and neural networks. Because ABMs are not restricted to rational behaviours but to learning and adaptation, there may be various ways to implement agent's behaviours. Holland and Miller (1991) note that there is usually one way to be fully rational, but many ways to be less rational. Tesfatsion (2002) argues that there is not a single algorithm that performs best in all situations, nor does any algorithm match the observed human decision-making behaviour under all conditions. The author also indicates that a better way to proceed is to let agents to "learn to learn" by evolving a repertoire of behavioural rules and modes activated depending on the situation, incorporating ideas from artificial intelligence.

In a very relevant work that compares top-down and bottom-up macroeconomics models, De Grauwe (2010) explains that in top-down models the agents fully understand the system, looking to optimize their welfare. Conversely, in bottom-up models, the agents do not understand the whole picture and have access to limited information. According to the author, while in the top-down models the business cycles can be exogenously driven (productivity and preferences shocks followed by a transmission lag to output and inflation), in bottom-up models the business cycles movements have a large endogenous component. Agents are willing to learn from their mistakes and they act on a trial-and-error learning process, leading to waves of optimism and pessimism, reflecting the difficulties agents face understanding the economic reality.

According to Tesfatsion (2002), the defining characteristic of agent-based models is their constructive grounding in interactions of autonomous adaptive agents. These agents are constrained by initial conditions set by the modeller, but the dynamics of the system are governed by agent-agent interactions (and not by exogenously imposed systems of equations). The state of the economy in each point in time is given by the varying internal attributes of the individual agents that currently populate the economy. In other words, an adaptive process emerges from the model dynamics, built on agents' characteristics, actions and their relationships. In contrast to traditional top down DSGE models, where equilibrium is usually an assumption, in ABMs the aggregate behaviour will emerge from the system dynamics. Comparably to an actual society, the agents interact with each other through social networks, influencing the other agents' behaviours and learning from these interactions. The way the agents interact with each other, and their environment supports the modelling of the additional characteristics of the system (e.g., bidding systems or auctions, competition for scarce resources, geographical constraints, etc.) and the observation of the collective behaviour originated from these interactions.

ABMs have been employed in different areas of science, including economics and social sciences, financial markets and biology, among other sectors. These models are particularly interesting for modelling complex systems from which outcomes emerge from interactions of diverse and interdependent agents. Through simulations, it may be possible not only to verify hypotheses but also to recognize outcomes not identified a priori. Pros and cons of ABMs in economics will be explored in the next section.

1.2 Pros and Cons of Agent-Based Models in Economics

ABMs bridge a gap between micro foundations and the macro situation, allowing the development of models without necessarily assuming optimization or equilibrium [Hamill and Gilbert (2016)]. ABMs can easily handle complexity and heterogeneity engendering collective behaviours in dynamic systems. In addition, ABMs allow the integration of behavioural aspects in models. These are very desirable features for modelling the dynamics of crisis scenarios, markets booms-busts and situations usually identified as fat tails or outliers, which cannot be completely explained by traditional models.

While these models allow for extended flexibility that complement classical models, one of the major challenges of ABMs identified by Gräbner (2016) is the propensity to overparameterization. Models may become extremely complicated, hard to review and discuss. Adding variables, processes and methods to perfectly fit data may create a system that memorizes data but would not be able to create good predictions. Good ABMs should increase the transparency in a study (and not the opposite). Model validation may not be as straightforward as in classical models. It is therefore advisable to publish the ABM code, so assumptions and replicability of the study can be verified. Another challenge identified by Hamill and Gilbert (2016) is the lack of standardization. Since the ABM allows modellers to move away from optimization, the vast possibility of behavioural rules that can be applied to agents' decision-making may have significant implications. Since this is a relatively new field, there is a need to develop on agreed assumptions and accepted agents' behavioural rules. The high dependency on micro data for the model calibration may also be a challenge for ABMs.

Gräbner (2016) identifies a very favourable aspect of ABMs: supporting better policy advice. Because ABMs provide a multidimensional perspective and have the possibility of showing how results emerge (constructive character), their ability of simulating potential effects of public policies is an important advantage of these models. The possibility of a simulation of the emergent behaviours on different points in time (step by step with time varying parameters) is a crucial advantage of ABM's in policy making, as elucidated by the *Lucas critique*.

Lucas (1976) concluded that, since the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes relevant to the decision makers, any change in policy will systematically alter the structure of econometric models. In other words, predicting the effects of a change in economic policy based on highly aggregate historical data would not be possible since agents would change their behaviour when new policies are introduced. While ABMs can be vulnerable to the Lucas critique, the adaptability of ABMs would allow the model dynamics to be altered following the introduction of an exogenous factor (e.g., a new policy introduction), with the possibility of following how the emergent behaviour is developing iteratively. Turrell (2016) observes that in future, additional artificial intelligence techniques could be applied to make agents respond realistically to new circumstances, making them more Luca's critique proof.

In a recent article, Haldane and Turrell (2019) observe that no model is fully Lucas critique proof. This is a matter of degree. In order to make models robust to changes in policy, traditional modellers seek to build the macroeconomic behaviour from microfoundations. The authors explain that these microfoundations are the aggregation of the individual actions of self-interested agents (usually looking to rationally optimize their utility) which are less susceptible to change. The authors observe that there are limitations to this approach, as heterogeneous micro behaviours can generate complex and non-linear emergent macro-outcomes. In addition, it is known that humans do not act rationally and homogeneously optimizing their utility. Haldane and Turrell (2019) argue that a plausible alternative is to employ empirically observed behaviours among agents. These behaviours usually involve heuristics and bounded rationality. The aggregate behaviour is often fat-tailed and emergent.

ABMs are particularly suited for this task, as they explain the evolution of a system by simulating the behaviour of each individual agent and then combine these individual and heterogeneous micro behaviours to generate an aggregate view.

ABMs allow the flexibility of creating models more Lucas critique proof and the incorporation of heterogeneous behavioural finance aspects. They allow the possibility of observing how a situation unfolds from the simulation of agents' endogenous behaviours and not only from exogenous shocks. Yet, there are still barriers to ABMs adoption. Haldane and Turrell (2019) identify that they are relatively recent, and therefore have not received the same investment as traditional models. In addition, the lack of a bridge to existing models and the necessity of some programming languages familiarity are also challenges. ABMs are more widely spread and accepted in areas where they were able to successfully explain phenomena that could not be explained by traditional models.

Pros and cons considered, ABMs offer a bottom-up perspective of the economy that complement the traditional top-down models. According to Turrell (2016), they are an important tool for understanding markets and provide a unique platform for augmenting policymakers' judgments about the economy. While this new field is still developing, ABMs have been gaining more and more attention as they are being applied to different areas of the economy, from consumer demand simulations to housing markets. The next section will focus on ABMs of the housing market.

1.3 Agent-Based Models of the Housing Market

The importance of the housing market is not limited to the proven effects it has on the rest of the economy. A home is usually the most significant asset a household will acquire, usually through credit. The real estate market influences the perceived wealth of households and their subsequent consumption decisions. Access to a safe home, independent of housing tenure (i.e. tenancy or owner-occupancy), often represents a turning point for households to prosper. On the other hand, housing market distortions, usually driven by behavioural factors, may result in widened inequality gaps or even affordability crisis. In addition, Crowe, Giovanni, Igan, and Rabanal (2013) observe that more than two thirds of the systemic banking crises for which housing data is available were preceded by boom-bust patterns in the housing market.

Since the housing market is subject to economic cycles, often strongly affected by behavioural aspects, the use of agent-based models (ABMs) may offer a very appropriate complement to existing classic DSGE models, in special for understanding the situations not completely described by traditional models and where heterogeneity may play an important role. ABMs allow considering heterogeneous observed behaviours in the model, not necessarily being restricted to rationally optimizing agent's utility. ABMs also allow the study of emergent outcomes from endogenous factors, including boom-bust cycles, as demonstrated by Ge (2017).

While very relevant work has been developed on ABMs focusing on specific housing markets, this approach is not yet widely and consistently applied. This is a relatively recent field and there is an opportunity for additional research, which may lead to promising advances, particularly in the area of informing policy making. Some of the models identified in the review of the literature target the following housing markets:

- United Kingdom [Baptista, et al. (2016)]
- England [Gilbert, Hawksworth, and Swinney (2009)]
- Denmark [Carstensen (2015)]
- United States [Ge (2017)]
- Washington, D.C., United States [Geanakoplos, et al. (2012)]
- Eugene-Springfield, United States [Waddell (2002)]
- Toronto-Hamilton, Canada [Rosenfield, Chingcuanco, and Miller (2013)]

Each model studied has specific characteristics and underlying goals related to the targeted housing market. While some models focus on an entire country aggregate market, other models look specifically at a region. Usually regional models are richer in details, while models applied to broader regions provide a better macroeconomic view of market dynamics.

The United Kingdom housing market was the object of study of a very rich ABM developed by the Bank of England. The model designed by Baptista, et al. (2016) focus on informing macroprudential policies. It includes heterogeneous agents representing households, mortgage lenders and a central bank. The agents representing households can be described under four different types: renters, first-time home buyers, home movers and buy-to-let landlords. The model calibration was done with a large set of micro data, including household surveys and housing market data. Households have properties (e.g., age, income) and their actions are based on defined decision algorithms. The agents representing the banking sector and the central bank incorporate the concepts of loan-to-income (LTI) and loan-to-value (LTV) affordability constraints to ensure a household has enough income to pay for its mortgage(s). The concept of interest-cover-ratio (ICR) is also built in to ensure that buy-to-let households can continue to afford mortgage payments if costs increase or if their rental income declines. This very comprehensive model was validated using Monte Carlo simulations to generate a distribution of the applicable variables. Then, empirical distributions were compared to those produced by the model, in addition to an analysis on the relationship of variables. The realization of experiments allowed verifying that the model was able to offer a better understanding of the effects of macroprudential policies in the UK market.

Still in the UK, an ABM proposed by Gilbert, Hawksworth, and Swinney (2009) studied specifically the English housing market. This model, which includes agents representing buyers, realtors and sellers, was built in NetLogo [Wilensky (1999)] using a 50 by 50 grid to represent a town (the environment where the agents will interact) with parameters that can be adjusted during the simulation to calibrate exogenous factors. This pioneer model allowed reproducing some of the main characteristics and interactions of the housing market and the investigation of shocks with a simple, yet very informative model. The ABM and code are available for download [Gilbert, Hawksworth, and Swinney (2008)], allowing for additional review of the assumptions and the design of the model.

The Denmark housing market was the object of a study developed by Carstensen (2015). An ABM was built using object-oriented programming language to analyse the asymmetry of exogenous shocks: interest rate and income. This ABM allowed the observation of the endogenous dynamics of prices and the role of macroprudential regulation. In this ABM, 5000 household agents with properties defining their age, income, moving probability and information of the market development interact continuously. Households' preferences are modelled following a Cobb-Douglas function that reflects the empirical share of total consumption. This allows agents to choose between allocating resources for housing (influencing the housing tenure and quality of the dwelling) or non-housing goods consumption. The probability of moving from a rental to a freehold or from one property to another is modelled based on the household age, income and on a market-specific component. Prices are set by owners based on a sample of similar houses sold in the neighbourhood with similar quality index in the last periods. In this model, the prices are automatically marked down if the house is not sold in three months. A single bank manages lending of households, following a given interest rate and borrowing criteria, which includes a debt to income (DTI) constraint. Monte Carlo simulations indicated that more flexible DTI limits on mortgage loans resulted in greater fluctuation of the housing prices.

An important aspect of modelling with ABMs is the model validation. As previously explained, this step may not be as straightforward as in classical models. While some model programmed assumptions and behaviours need to be verified by analysing the ABM source code directly, the validation of the model generated data in contrast to empirical data can be done comparably to traditional models. As described by Baptista, et al. (2016), Monte Carlo simulations are often used to generate a distribution of the applicable model variables. Carstensen (2015) explained in detail the steps applied to ensure the model was stable before it could be validated against empirical data. The author used a Wald-Wolfowitz test (or runs-test) to verify the stationarity and the ergodicity of the time series generated by the ABM. Ensuring the relevant statistics were stationary time series allowed establishing statistical equilibrium. The validation of ergodicity is done to confirm that the effect of initial model conditions eventually vanishes. For this specific ABM of the Danish housing market, the author noted that the non-parametric test for ergodicity indicated that the initial state of the model could have some effect on price levels, not impeding though the price levels variation analysis.

The United States' housing market was studied by Ge (2017). Following the 2007-2009 financial crisis, this ABM dynamics demonstrate how endogenous factors could lead to increased volatility in the housing market. This model proposes five types of agents: real estate agents, developers, banks, buyers and homeowners. It uses a simplified 5 by 5 grid with 25 regions as the environment landscape. It introduces the concept of geographic neighbourhood quality index, which is assigned to each region, as explained by Ge (2013). The household's decisions follow a classic approach (maximization of the utility function). The experiments run were able to demonstrate that housing bubbles (sharp price rises immediately followed by collapses) can be generated uniquely by endogenous factors, namely lenient lending and speculation. Although an exogenous shock could increase volatility, the results showed that housing prices can rise and collapse endogenously without any external shock.

Also following the 2007-2009 financial crisis, the Washington, DC area was the object of study by Geanakoplos, et al. (2012). In a remarkable paper focused on the systemic risk of the housing market, a very detailed ABM of the area was built with the objective of retrospectively understand the boom and bust of the 1997-2009 market in the US. This housing market model was based on a previous model developed by Geanakoplos and his team in the 1990s on mortgage prepayments. Since mortgage prepayments change the cash flows and valuations of securitization products, Wall Street investment banks have been using ABMs to monitor risk and prepayment forecasts. The mortgage prepayment model, in which the subsequent housing market model is based, tries to predict the prepayment of mortgage pools (aggregation of individual mortgages that are sold as securities by lenders). Instead of working with aggregate predictions, the model focuses on the individual homeowner and produces aggregate forecasts by adding up the behaviour of the individual agents. Agents are modelled with heterogeneous properties representing the cost of prepayment, and an alertness factor representing the awareness of prepayment advantages and a turnover rate (possibility of selling the house). By allowing the model to run retrospectively, it was possible to fit the historical data and run reasonable conditional predictions. Nevertheless, with the introduction of the cash-out refinance possibility in the 2000s, the mortgage prepayments ABM became obsolete and had to be updated to consider the house prices appreciation and the new behaviours introduced by the new prospect. The authors comment that one of the criticisms of ABMs is that behavioural rules eventually become inappropriate as the world changes. This is a good link to the Lucas critique described in the previous section, as this illustrates a model vulnerability to time varying parameters.

The housing market ABM that was then developed by Geanakoplos, et al. (2012), as a progression of the mortgage prepayment ABM, applying a similar approach but this time with a more audacious objective focused on the housing market cycles. The intent was to represent every housing unit in the Washington, DC area (owner-occupied and rentals), which also incorporated agent properties for income, wealth, age, marital status, among others. The model relies on data for household demographics, economic conditions, housing shocks, loan characteristics and household market behaviours. During the simulations, new households are formed and decide to rent or buy a property. Houses are also put on the market when households move, foreclose or die, for a price a little above the recent sales. As described by Carstensen (2015), who was inspired by this ABM, the asking price is marked down if the house is not sold in a few months. The model considers that the households will bid on the highest valued property they can afford, considering they will spend a third of the income in housing and satisfy a defined loan-to-value (LTV). By freezing the LTV and the interest rate variables, the model allowed authors to observe by simulation that leverage (and not interest rates) played an important role on the 1997-2009 boom and bust of the US housing sector, which preceded the 2007-2009 financial crisis.

An alternative approach focused on urban planning is proposed by Waddell (2002), who developed a comprehensive simulation system named *UrbanSim*. This approach consists of a microsimulation platform that includes a very sophisticated group of models for addressing different aspects of urban planning. It has been already applied to metropolitan areas across the United States and internationally. Waddell (2002) explains the architecture of the planning tool designed for modelling urban development, land use, transportation and environment planning. The article also describes the application and validation of the model in the Eugene-Springfield area in the US. *UrbanSim* goes beyond modelling the housing market, as it accounts for additional urban planning issues such as transportation and land use. This urban simulation system was designed as open-source software and is composed of several specialized modules that interact with each other reflecting the key behavioural choices of households, businesses, developers, governments and their interactions with the real estate market. Waddell (2002) explains that the model represents demand for real estate at each location and the choice processes that influence patterns of urban development and real estate prices.

The models that compose *UrbanSim* employ different approaches, from aggregate top-down modelling to location-based bottom-up modelling. Locations are represented in grid cells superposed to a region ortho-photograph. This allows not only modelling specific geographical locations, but also identifying the real estate density. The system has a centralized data set from where data for households, jobs, land and real estate are made available. The application to the Eugene-Springfield area included 15,000 grid cells of 150 to 150 metres and extensive data about the region. For this area, after calibrating and running the simulation for a 15-year period, the results correlated well with the empirical data. While *UrbanSim* cannot be considered a simple ABM of the housing market, it implements bottom-up approaches in some of its composing modules (e.g., the real estate developer module) and it encompasses real estate prices. One of the identified development priorities included adding more behavioural rules representing the roles of landowners, lenders, investors and specialized developers.

A comparable model focusing on the Toronto-Hamilton area in Canada is presented by Rosenfield, Chingcuanco, and Miller (2013). The housing market dynamics is built as a module of an existing agent-based model named ILUTE (Integrated Land Use, Transportation, Environment). ILUTE is a detailed ABM that focuses on demographics, travel behaviour and economic structure over time of a defined urban region. The housing proposed module, named HoMES (Housing Market Evolutionary System), focuses on the owner-occupied housing market, with the implementation of a bid-auction process for the formation of the housing prices. Market entry is determined by residential stressors, which include family changing composition, changes in employment and surrounding economic conditions. The model seeks to determine how households choose potential homes, considering dwelling preferences, location and property valuation. The market clearing is done via an auction process where buyers willingness to pay (WTP) and dwelling utility define transactions price.

Different types of ABMs of housing markets have been proposed, with various purposes and from different perspectives. These models contemplate some relevant concepts to be explored in an ABM of the Canadian housing market. In the next section, a brief analysis of the Canadian housing market main characteristics will be presented.

1.4 The Canadian Housing Market

Canada's housing market has grown significantly over the last years, stimulated by economic development, population growth and favourable credit conditions. Unlike many other countries, housing prices generally remained high during the 2007-2009 financial crisis. Long-term averages of Canadian house prices relative to income (price-to-income) and rents (price-to-rent) are among the highest of OECD countries [Cheung (2014)].

Even with an aggregate notable growth, it is important to observe that housing prices dynamics are not homogeneous. While some metropolitan areas show overvaluation signals, other areas present less evidence of vulnerability. As a rich and diverse country, Canada housing market is affected by regional idiosyncrasies. In an empirical analysis of the main Canadian cities housing prices, Allen, Amano, Byrne, and Gregory (2007) found that the main cities' real-estate prices are only weakly correlated in the long run. Because of the lack of cointegration (long-run relationships of the house prices) among cities, it is recommended to consider local factors for understanding the Canadian housing market cycles. In addition, in a study about common cycles in Canada, Wakerly, Scott, and Nason (2006) found that Canadian regional business cycles are driven by a set of disaggregate propagation and growth mechanisms, with asymmetries in the volatility, correlation structure and persistence of the regional cycles.

In another very relevant study targeting regional Canadian housing prices, Lin and Fuerst (2014) found that the majority of provincial housing markets in Canada exhibit some characteristics of stock indices, namely volatility clustering, positive risk-return and leverage effects. Despite having low liquidity and high transaction costs (in opposition to stocks, which have high liquidity and low transaction costs), most Canadian provinces housing markets exhibit periods of high volatility followed by higher volatility and periods of low volatility followed by lower volatility (volatility clustering), in addition to positive risk-return and leverage effects. The phenomenon is heterogeneous, with more densely populated provinces showing stronger volatility clustering. This could be explained by the fact that, in areas more populated the transmission and persistence of information by local citizens would be more efficient than in areas with less population, directly influencing the local housing market in the short run. The authors also found evidence that most populated areas may be a leading factor influencing the rest of the adjacent housing markets.

The analysis of housing prices development frequently applies two commonly used ratios: price-to-rent and price-to-income. The price-to-rent ratio is comparable to the price-to-earnings multiple for stocks and is intended to reflect the cost of owning versus renting. Conversely, the price-to-income ratio measures the local housing costs relative to the local ability to pay [Himmelberg, Mayer, and Sinai (2005)]. Although useful, these indexes cannot be used as simple indicators of overvaluation or comparison of housing markets, as they may be biased by a series of local conditions. Cheung (2014) explains that price-to-rent ratios may be affected by rent controls (present in some form in the Canadian provinces of Ontario, Quebec, British Columbia, Manitoba and Prince Edward Island) restricting rises in rents. In addition, price-to-income ratios are usually based on average measures, ignoring the fact that homeowner's income level is usually higher than the average population. The author observes that these metrics may also ignore the effects of lower interest rates increasing home ownership affordability.

Himmelberg, Mayer, and Sinai (2005) argue that price-to-rent and price-to-income measures are inadequate to assess housing markets and propose an alternative analysis, taking into consideration the financial return associated with the property ownership, including the cost that would be incurred to rent an equivalent property, the opportunity cost of the capital invested in the house and additional factors, such as risk, tax benefits, property taxes and maintenance, and the anticipated capital gains from owning the property. This annually calculated cost of ownership could then be compared to local rental costs and local income levels. The authors conclude that housing prices dynamics are a local phenomenon and that changes in underlying factors such as long-term interest rates, expected inflation, expected prices appreciation and taxes affect cities differently. Therefore price-to-rent and prince-to-income ratios of different cities should not be compared directly. In addition, authors concluded that there was little evidence of housing bubbles in almost any of the markets studied [Himmelberg, Mayer, and Sinai (2005)]. The paper studied various metropolitan areas of the United States and was published just before the 2007-2009 financial crisis.

A major vulnerability noted across Canada is the increase of household debt levels. According to an OECD study of the Canadian housing market by Cheung (2014), household debt started trending up in the mid-1980s, from a level of 60% of disposable income. It reached 166% of disposable income by mid-2013, when that paper was written. Five years later, by mid-2018, the Canadian debt-to-income ratio reached 175% of the disposable income, with some cities (Victoria, Vancouver and Toronto) exceeding a debt-to-income ratio of 200% in 2016 [Statistics Canada (2019)], mostly driven by the accumulation of mortgage liabilities.

In a recent study about the indebtedness and wealth of Canadian households, Gellatly and Richards (2019) found that while the debt-to-income levels of the US households decreased by 25% since the 2007-2009 financial crisis, Canadian households' debt-to-income increased by 20% since 2007. At the same time, the appreciation of the value of housing assets has strengthened these households balance sheets, by augmenting households net worth. Consequently, the debt-to-asset ratios in Canada decreased since the last financial crisis. With households strongly leveraged, especially in some metropolitan areas, and because economic growth has been driven by household spending (in opposition to investment spending and exports), the household indebtedness has garnered increasing attention in recent years.

According to Cheung (2014), rising interest rates, job losses or a shock causing a significant house price correction could place a considerable strain on the households and banks balance sheets in Canada. Differently from the US, where mortgage rates are usually locked in for 30 years, Canadian mortgage interest rates are negotiated in periods of 6 months to 5 years, through an amortization period of usually 25 years. Canadian households are therefore exposed to a risk of higher interest rates at the renewal of their mortgage contracts. From the income perspective, Crawford, Meh, and Jie (2013) note that increases in mortgage arrears rate are closely related to loss of employment and income. The higher the debt-service burden of households, the more vulnerable they are to adverse shocks.

The Canadian housing market has also some remarkable differences if compared to the neighbour US housing market. The majority of home loans are insured against default due to stricter regulation requiring federally regulated financial institutions to insure all mortgages with loan-to-value (LTV) greater than 80%. Mortgage insurance in Canada is provided by Canada Mortgage and Housing Corporation (CMHC), Sagen (previously Genworth Financial) and Canada Guaranty. CMHC is an important mortgage insurer in Canada, and is also Canada's national housing agency, a crown corporation with the mandate to promote housing affordability and a stable housing finance system. CMHC also manages mortgage funding activities through securitization programs: National Housing Act (NHA) Mortgage-Backed Securities (MBS) and Canada Mortgage Bond (CMB). In addition, most Canadian financial institutions, including insurance companies, trust companies, loan companies and pension plans are supervised and regulated by the Office of the Superintendent of Financial Institutions (OSFI).

Crawford, Meh, and Jie (2013) explain that the housing finance system in Canada is composed of three types of institutions: mortgage originators (lenders), mortgage insurers and suppliers of funding. Most of the mortgage lenders are chartered banks, with lending dominated by the five largest Canadian banks. Trust and mortgage loan companies, credit unions and caisses populaires, life insurance companies, pension funds and non-depository credit intermediaries also participate in the market, but in a smaller scale. Since about 80% of mortgages are originated by lenders supervised by OSFI, there is a stronger regulation on institution risks and risk management. For example, in 2012 OSFI issued the guideline B-20, indicating that the loan decision should be based on the borrower demonstrated willingness and capacity to make debt payments on a timely basis and not only on the value of the collateral housing asset. This guideline has been revised in 2017, with the introduction of the requirement of borrowers to pass an interest rate stress test, among other updates to reinforce a strong and prudent regulatory regime for residential underwriting in Canada [OSFI (2017)].

The resilience of the Canadian housing market during the last financial crisis was supported by strong financial oversight and government intervention [Cheung (2014)]. For example, in 2008, the Canadian government provided mortgage lenders with an additional source of liquidity during the crisis by buying NHA MBS from financial institutions through the Insured Mortgage Purchase Program (IMPP). Cheung (2014) observes that after loosening a series of lending restrictions from 2003 to 2007, starting in 2008 a series of macroprudential measures have been implemented through tighter regulations on government backed mortgage insurance to mitigate the risks to the financial stability. Crawford, Meh, and Jie (2013) observe that in addition to the strong supervisory framework and minimum qualifying standards for mortgage insurance, other provisions reduce the overall risks to the financial system and the housing market from a potential correction in housing prices. These provisions include the non-deductibility of mortgage interest payments from the taxable income and also Canada's recourse laws, which reduce the incentive for households with negative housing equity to default.

Despite the resilient housing market, supported by strong regulation and availability of tools for government intervention, housing affordability has worsened in Canada. Following the latest years housing prices appreciation in large Canadian metropolitan centres, housing affordability is becoming a challenge, especially for those in the lower income brackets. In 2011, one third of all Canadian households lived in unaffordable housing (spending more than 30% of their pre-tax income on shelter costs). In the same year, 13.2% of households were in core housing need, meaning their dwelling was below housing standards, in addition to income being insufficient to obtain acceptable housing [Cheung (2014)]. In 2016, the number of Canadian households in core housing need increased to 13.6%. The standards that define acceptable housing are: (1) adequate housing not requiring any major repairs; (2) affordable housing costing less than 30% of total pre-tax income; and (3) suitable housing with enough bedrooms according to National Occupancy Standard (NOS) requirements [CMHC (2018)]. In response, the latest Canadian budgets have introduced measures to improve housing affordability, including the delivery of the National Housing Strategy (NHS), a 10-year plan to help Canadians access housing that meets their needs and that they can afford [Government of Canada (2019)].

Nevertheless, the shock caused by the COVID-19 pandemic contributed to widening housing affordability issues in Canada by creating a situation of constrained housing inventory and record low interest rates, which contributed to significant surges in housing prices [Government of Canada (2021)]. The latest budget proposes measures to try to restore housing affordability in Canada by focusing on affordable housing supply investments (i.e., double housing construction over the next decade), by creating fiscal incentives to first-time home buyers and by curbing foreign investment and speculation [Government of Canada (2022)].

While regional markets may be subject to local factors and regulation, they also respond to federal macroprudential policies, which may generate different responses in different regions. This characteristic makes the Canadian housing market a unique subject for an ABM. Since ABMs support heterogeneous agents and can provide a multidimensional model perspective, the local characteristics of regional markets can be maintained and observed during a simulation. Therefore, an ABM of the whole Canadian housing market would potentially provide an opportunity to observe the heterogeneous dynamics of the regional markets emerging in a countrywide model with the opportunity of studying not only the aggregate behaviour but also the sectorial unfolding of outcomes. Because ABMs simulate emergent behaviours constructively, in different points in time iterations, such a model would be extremely helpful in informing policy making, targeted on promoting housing affordability and maintaining a stable housing finance system in Canada.

CHAPTER II

OBJECTIVE

The objective of this work is to create an agent-based model of the Canadian housing market using the ABM proposed by Gilbert, Hawksworth, and Swinney (2009) as a baseline. This specific model was selected as a starting point for a series of reasons, notably because of its source code availability, its educational value and its relative simplicity in terms of programing since it was developed using NetLogo [Wilensky (1999)].

NetLogo is an integrated development environment for agent-based modeling first created in 1999 and inspired by the programming language Logo. It is open source and a series of models from different domains are available in an accessible library. It offers a good graphic user interface that facilitates inputting variables and parameters and observing results. It is an established multi-agent programmable modeling environment used worldwide for academic purposes, which is regularly enhanced with additional features and extensions.

The ABM proposed by Gilbert, Hawksworth, and Swinney (2009) studied the English housing market in a simplified representation that included buyers, realtors and sellers. It incorporates Loan to Value (LTV), interest rate and inflation, in addition to other variables and parameters. The objective of this work is to understand the original baseline model, briefly describe it and then update it (extending it) to study the Canadian housing market. One of the key proposed extensions is the possibility of reading empirical data related to the economy (e.g. mortgage interest rate, inflation rate and median household income) to better calibrate the model for a specific economy. This is a feature is not available in the original ABM.

Although the Canadian housing market cannot be considered homogeneous due to regional idiosyncrasies, for the purposes of this work a generalized approach will be used. The objective is studying the Canadian housing market as a whole (at the country level, not focusing on specific regional aspects). Even though this is a simplification, a macro level model would still be pertinent to study shocks that affect the market entirely, the COVID-19 pandemic being a recent one.

The objective is to create a model that could simulate housing prices fluctuations based on the variation of the mortgage interest rate, inflation and disposable income, with the possibility of also adjusting parameters during the simulation to observe the impact of these changes. The initial goal is to allow the ABM to generate median house prices based on empirical inputs and then compare the generated median house prices and house price variations to housing prices empirical data. Once a validation that the model is able to generate acceptable outputs is done, simulations can be run to observe impact of changes to some parameters (such as leverage). Mostly the objective of an ABM is not producing forecasts, but to build a model that could potentially inform macroprudential policy making by allowing a bottom-up simulation of behaviours in response to environmental changes.

CHAPTER III

DATA DESCRIPTION

The data included in this research will be used to calibrate and validate the Canadian agent-based model, reflecting the characteristics of the Canadian housing market. Although the baseline model proposed by Gilbert, Hawksworth, and Swinney (2009) does not incorporate empirical data, the proposed Canadian version of the ABM reads 3 time series of data representing: (1) the yearly average of 5-year conventional mortgage rate; (2) the yearly average inflation rate; and (3) the yearly average of households' disposable income. Based on data availability, 48 data points from each time series were selected, from 1975 to 2022. In addition to these series of data, a time series of "*Residential Property Prices in Canada*" was used to validate the output of the ABM. This data is not used as an input variable to the model, and it is only read and incorporated to the user interface to allow a comparison of the agent-based model data with the real-world data during the simulation.

The next sections will describe each series of data and any conversions applied to the original data from 1975 to 2022 to generate the time series employed during the simulations.

3.1 Yearly Average of 5-Year Conventional Mortgage Rates in Canada

The Bank of Canada (BoC) publishes a weekly time series with the interest rates posted by the major chartered banks in Canada for selected products [Bank of Canada (2022)], namely: prime rate, 1-year conventional mortgages, 3-year conventional mortgages, 5-year conventional mortgages, 1-year guaranteed investment certificates, 3-year guaranteed investment certificates, 5-year guaranteed investment certificates, 5-year personal fixed term, and daily interest savings for balances over $100,000 and non-chequable savings deposits.

As described by Cheung (2014), residential mortgages in Canada have typically consisted of fixed-rate loans amortised over 25 years and with terms varying from 6 months to 5 years. In addition, mortgage lending in Canada is concentrated with the major chartered banks and funded (to a great extent) from retail deposits [Cheung (2014)]. Based on this, the 5-year conventional mortgages time series of data was selected as the best representation of an aggregation of mortgage rates in Canada. This series of data is available since 1975, with 52 (weekly) observations per year.

Since the simulation will be run in yearly intervals, it was necessary to convert the weekly observations data to yearly observations from 1975 to 2022. In order to generate a yearly time series, a simple average was applied to the original weekly data, generating the average by year of the 5-year conventional mortgage rates, as demonstrated in *Figure 1 - Yearly Average of 5-Year Conventional Mortgage Rates in* Canada.

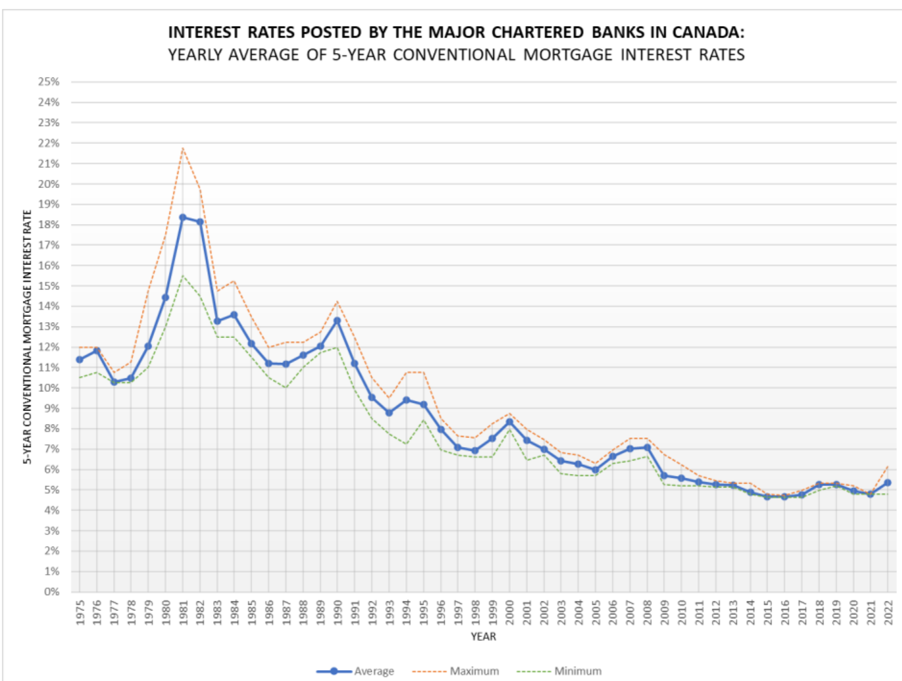

Figure 1 - Yearly Average of 5-Year Conventional Mortgage Rates in Canada

It is notable that in the most recent observations (in 2022) the mortgage rates started to increase, after a long period of sustained low rates that started just after the 2007-2009 financial crisis in the United States. Hence and due to its relevance, the data from the current year, which is still incomplete, will be included in the simulation. The *Table 1 - Yearly Average of 5-Year Conventional Mortgage Rates in Canada* in the appendix provides a detailed view of the calculated values for the mortgage interest rates in Canada.

3.2 Yearly Average of Inflation Rate in Canada

Statistics Canada regularly publishes a table with the Consumer Price Index (CPI) annual average, not seasonally adjusted [Statistics Canada (2022)]. The CPI is calculated for different product groups by year. In order to generate the yearly average inflation rate in Canada, the CPI variation year over year was calculated for the index of “all items”, which includes eight major components: “food”, “shelter”, “household operations, furnishings and equipment”, “clothing and footwear”, “transportation”, “health and personal care”, “recreation, education and reading”, and “alcoholic beverages, tobacco products and recreational cannabis” [Statistics Canada (2022)].

Data is available from 1914 to 2021. For this research, a subset of this data was used, starting in 1975, until the most recent calculated index for 2021 in this series of data. One additional data point was inputted for 2022, based on the latest monthly CPI data available, for July 2022 [Statistics Canada (2022)]. This created one notable increase in the variation of the inflation rate for the latest period, which is consistent with the current year (2022) latest monthly reports of inflation [Statistics Canada (2022)]. The resulting time series can be visualized in *Figure 2 - Yearly Average of Inflation Rate in Canada*. The *Table 2 - Yearly Average of Inflation Rate in Canada* is included and described in the appendix to provide visualization of the detailed data.

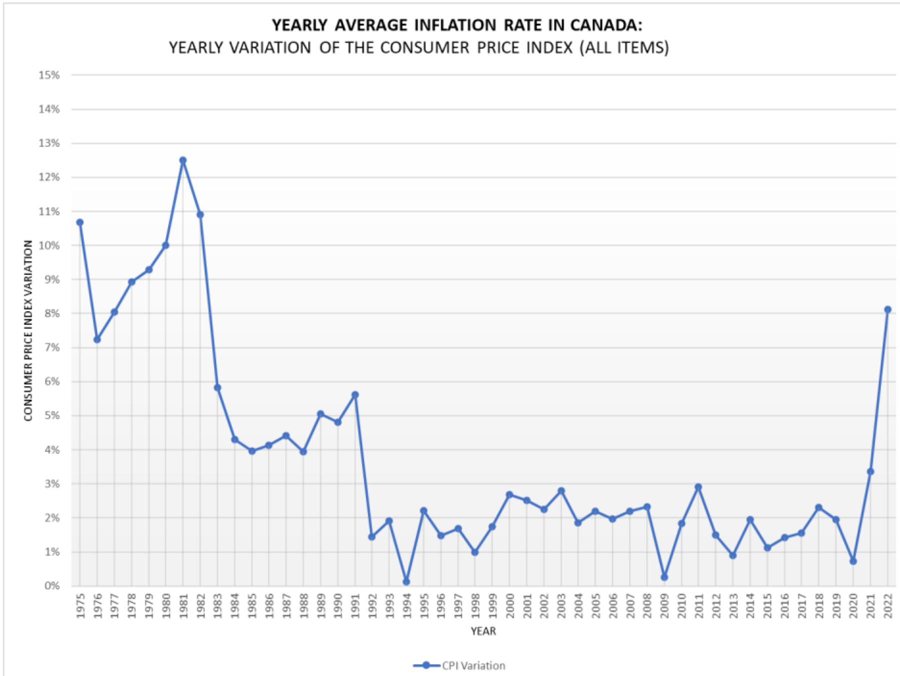

Figure 2 - Yearly Average of Inflation Rate in Canada

3.3 Yearly Average of Households Disposable Income in Canada

The data available from Statistics Canada on a table for market income, government transfers, total income, income tax and after-tax income by economic family type was used to estimate the median household disposable income in Canada. The median after-tax income for economic families and persons not in an economic family from 1976 to 2020 was read from the available time series, which includes the incomes in 2020 dollars [Statistics Canada, (2022)].

Since the ABM uses nominal values for simulations, the CPI was used to generate the nominal household disposable income for each year. It was applied to convert the original time series amounts in 2020 dollars to dollars of each year. Since data was not available for 1975, 2021 and 2022, the nearest data point available was used and converted to a nominal amount using the CPI for that year. The resulting time series is represented in *Figure 3 - Yearly Median Household Disposable Income in Canada*. *Table 3 - Yearly Median Household Disposable Income in Canada* in the appendix includes the detailed data and calculations.

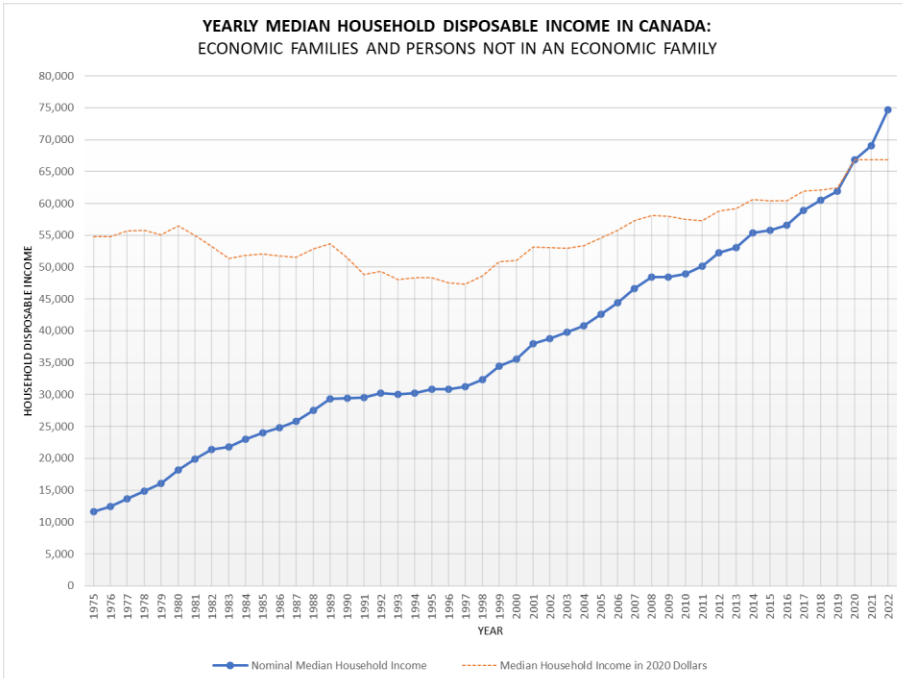

Figure 3 - Yearly Median Household Disposable Income in Canada

3.4 Residential Property Prices in Canada

The Bank for International Settlements (BIS) produces time series of residential property prices for various countries, including Canada, which can be accessed through the web site of economic data from the Federal Reserve Bank of St. Louis, the Federal Reserve Economic Data (FRED). The "*Residential Property Prices in Canada*" [BIS (2022)] and "*Real Residential Property Prices in Canada*" [BIS (2022)] index is referenced in 2010, with data from 1970 to 2022. A subset of this series, from 1975 to 2022, was used to calculate an estimation of residential property prices in Canada and the variation of prices for the period. This data, presented in *Figure 4 - Residential Property Prices in Canada* and *Figure 5 - Variation of Residential Property Prices in Canada* will only be used to validate the model against real world data. Calculation details are presented in *Table 4 - Residential Property Prices in Canada* in the appendix.

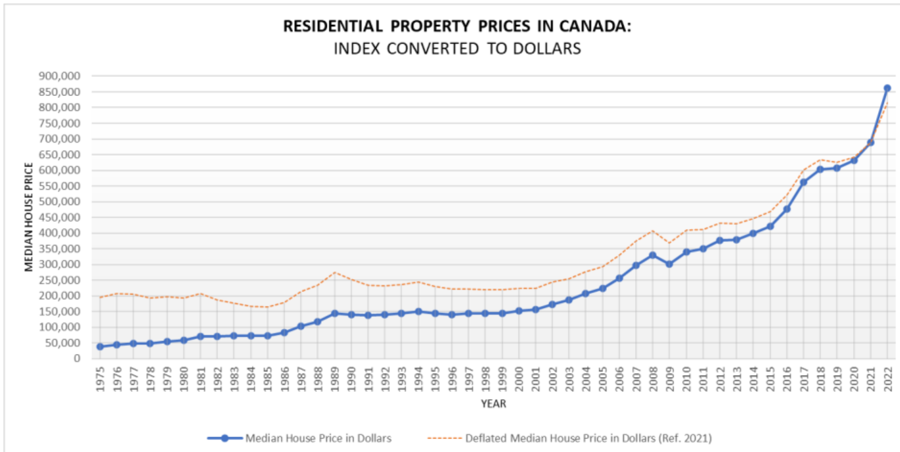

Figure 4 - Residential Property Prices in Canada

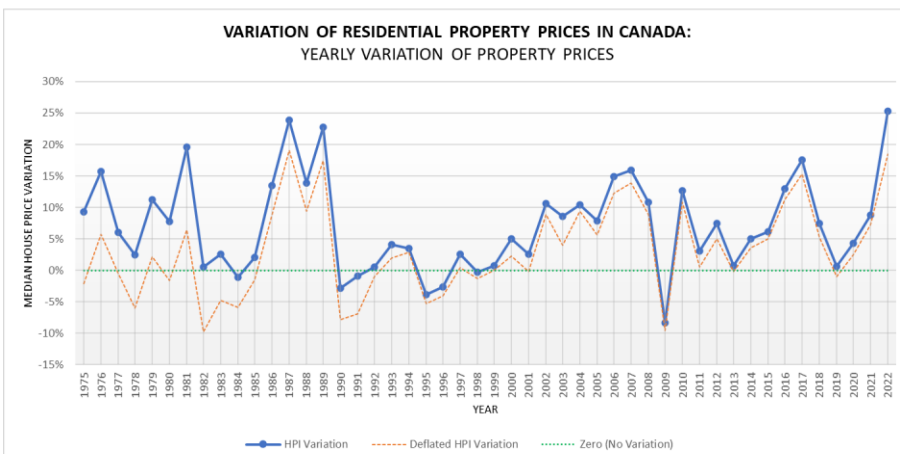

Figure 5 - Variation of Residential Property Prices in Canada

It is possible to observe the difference of the nominal and the deflated (with reference in 2021) residential property prices in *Figure 4 - Residential Property Prices in Canada*. In *Figure 5 - Variation of Residential Property Prices in Canada* the variation of the nominal and deflated prices can be observed. Since the model simulation generates nominal values for each year, the series with nominal prices and variation will be used as benchmark for validation.

CHAPTER IV

METHODOLOGY

This chapter will describe the methodology that was used to create an agent-based model of the Canadian housing market. The description of the baseline model, which was proposed by Gilbert, Hawksworth, and Swinney (2009) will be presented, followed by the description of updates and extensions introduced in the model to adapt it to the Canadian housing market. Finally, the methodology used to run and validate the updated model will be described. The objective of this work is to confirm that it is possible to simulate housing prices fluctuations based on empirical data from Canada, still consistent with the baseline model.

4.1 Base Model Description

The baseline model and NetLogo 5.3.1 were downloaded from the University of Surrey [Gilbert, Hawksworth, & Swinney (2008)] and NetLogo [Wilensky (1999)] websites respectively. In this phase of the research, NetLogo 5.3.1 (which is not the latest version available) was used for compatibility to the original model. Once installed, the model was run and the source code was examined, so the model could be understood and later extended.

The graphic user interface of the original model is shown in *Figure 6 - Baseline Model [Gilbert, Hawksworth, & Swinney (2008)]*. It contains various sliders and switches to update and modify the model variables and parameters, separated in different groups: macro-economy, owners, realtors and houses. The user interface also contains various output plots from where it is possible to verify and export the data being generated by the model. The main element of the interface is a grid of patches that represents the world where agents are interacting, as shown in the *Figure 7 - Detail of the Model Grid (Patches Representing the World)*. Green patches represent empty land and red patches represent houses. The darker the shade of red, the higher is the price of the house. Homeowners are represented by grey dots, which are plotted inside a red patch that represents a house. Red patches without a grey dot represent a vacant house. In this model, realtors are also represented by large yellow dots and a larger circumference representing the area of their influence. There are also three buttons to control the model: a "*Setup*" button to initialize the model, a "*One Tick*" button to run the model for one tick or period, and a "*Go*" button to run the model continuously.

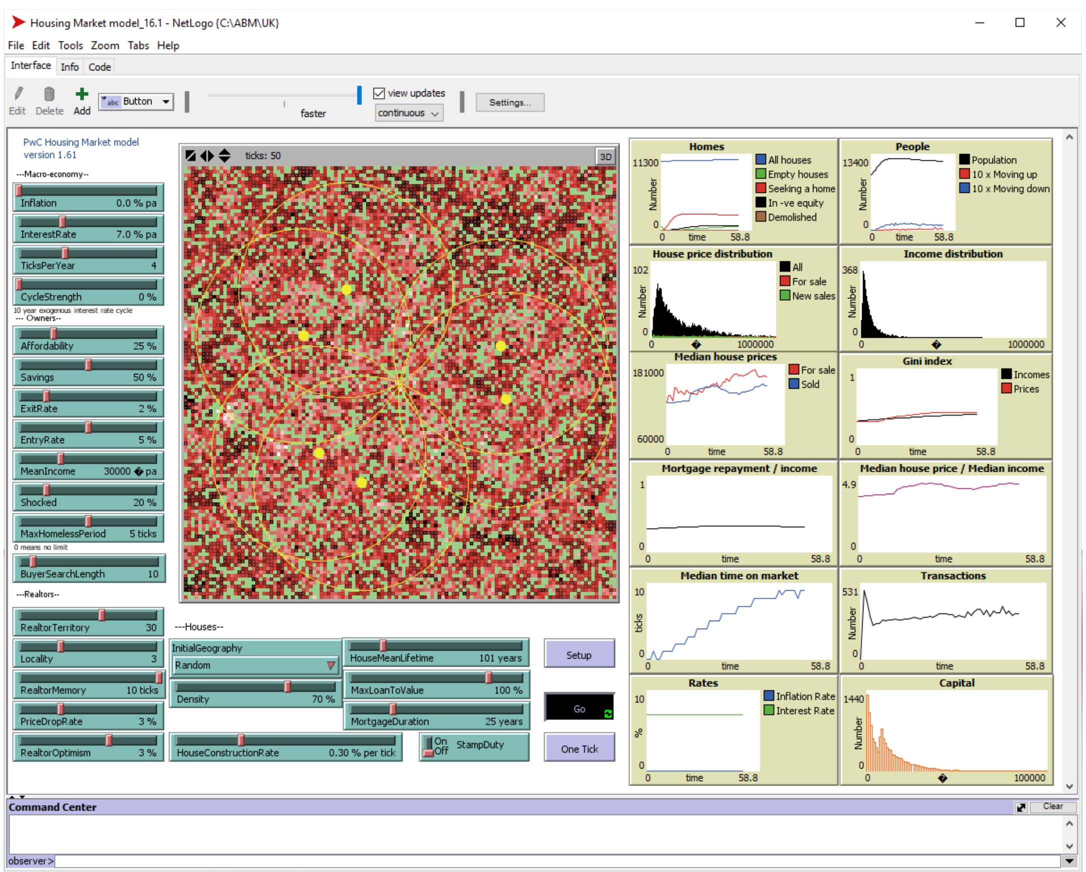

Figure 6 - Baseline Model [Gilbert, Hawksworth, & Swinney (2008)]

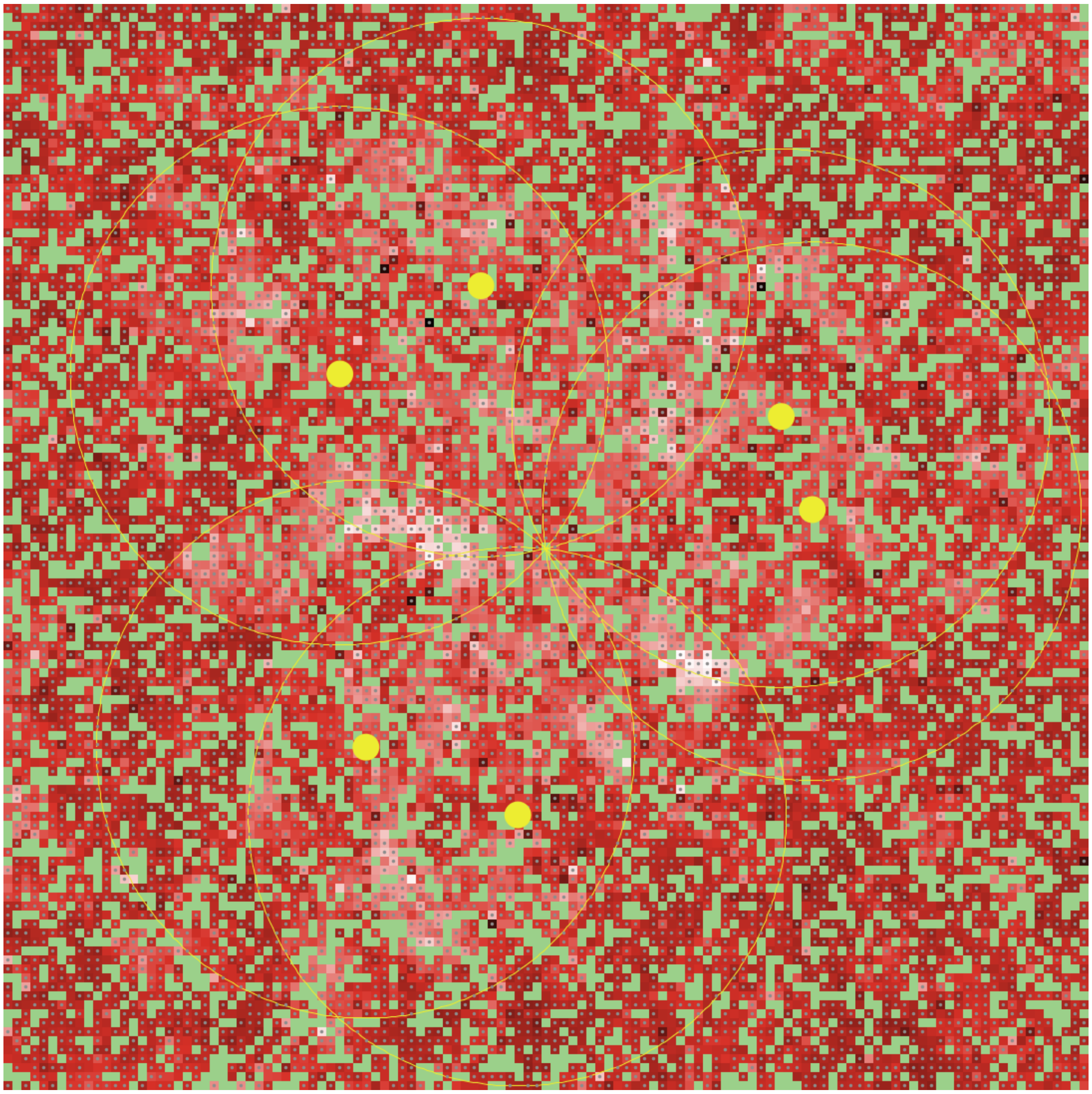

Figure 7 - Detail of the Model Grid (Patches Representing the World)

The original model proposed by Gilbert, Hawksworth, & Swinney (2008) does not read data from any external data sources. It uses the values configured in the user interface sliders to generate the data that will be used as input (including inflation, interest rate and mean income), as it will be described in more details later. In order to start a simulation, a setup procedure needs to be run, which is done by clicking the "*Setup*" button in the user interface. This will call a pre-programed "*setup*" procedure that initializes the time, which is measured in ticks in the NetLogo simulation tool. The procedure also sets up the variables and parameters to the baseline scenario. The model can be run continuously for 100 years (by clicking the "*Go*" button) or it can be run one tick at a time (by clicking the "*One Tick*" button). By default, one tick corresponds to a quarter of a year (total of 4 ticks per year). The model allows the user to adjust the number of ticks per year and other parameters during the simulation. The "*setup*" procedure also defines the initial LTV, interest rate, entry and exit rates (in the housing market), mean income, vacancy rate, number of realtors and maximum homeless periods. Once these and other variables and parameters are defined, the function plots the agents into the grid, representing houses, homeowners (buyers and sellers) and the realtors, along with their area of influence.

A simulation starts when the "*go*" procedure (which is called by the "*Go*" button in the user interface) is run. This procedure creates a basic loop to allow the model to run for 400 ticks (or 100 years). In the middle of the period, a scenario change is introduced (this could be a change in LTV, a decrease in interest rate, influx of more people, or a decrease in the mean income, depending on the scenario selected). Each tick iteration will trigger the "*step*" procedure (which could alternatively be directly called by clicking the "*One Tick*" button in the user interface).

The "*step*" procedure runs once for each period or tick. It allows the agents to respond to the environmental changes. Agents include (1) houses, (2) homeowners (or households) and (3) real estate agents with (4) records of sale. Houses may be occupied or empty and homeowners may be living in a house or looking for one. The real estate agents ensure records of sale transactions for their area of influence are kept, which will be used in valuations. The homeowners may contract a mortgage to acquire a house if they do not have enough savings. The loan details are configurable in the user interface sliders (including maximum loan to value (LTV) and gross debt service (GDS), referred as "affordability" in this version of the model).

In this model, an exogenous cyclical interest rate per period or tick (i_{tick}) is created varying around the mean of the value defined in the interest rate "*InterestRate*" (i) slider within a fixed period of 10 years and depending on the value defined in the "*CycleStrength*" (CS) slider. It considers how many "*TicksPerYear*" (T) are configured in the user interface (by default 4) to calculate the cyclical rate using a sine wave. The cyclical interest rate generated by the model can be represented by the formula:

$$i_{tick} = \frac{i}{T \times 100} \times \left(1 + \left(\frac{CS}{100}\right)\right) \times \sin\left(\frac{36 \times tick}{T}\right)$$

This generates interest rate cycles, following a sinusoid wave through the simulation. The generated interest rate is used for the calculation of mortgage payments, and consequently to determine which houses the buyers will be able to afford if a mortgage is contracted.

The buyer agents income (W) is assigned randomly using a gamma distribution, as described by Gilbert, Hawksworth, and Swinney (2009), using the values configured in the "*MeanIncome*" slider. In addition, incomes (W) are updated every tick based on the value configured in the "*InflationRate*" (I) slider, also considering how many "*TicksPerYear*" (T) are setup, as represented by the formula:

$$W_{tick} = W_{tick-1} \times \left(1 + \frac{I}{T \times 100}\right)$$

Another slider for "*Savings*" defines a proportion of homeowners' income that becomes savings for a future deposit. The slider "*Affordability*" allows the configuration of the maximum percentage of the agent income that can be used to service the mortgage payments, implying that a form of GDS ratio is also considered for house buying transactions.

In each step, an income shock is introduced for a percentage of the agents, depending on the percentage defined in the "*Shocked*" slider. It can be a positive (or a negative) shock of 20% more (or 20% less) income than before (the amount of the shock is fixed in 20%). Homeowners that have a mortgage have to pay interest and a part of the principal. Every tick, the mortgage is reduced by the amount of principal repayment. In addition, if owner-occupiers of houses are spending less than half the ratio defined in the "*Affordability*" slider on their mortgage repayments, they will want to move to a more expensive house (they will try to sell their houses and buy another one). Conversely, if owner-occupiers are spending more than twice of the ratio defined in the "*Affordability*" slider, they will want to move to a less expensive house (they will try to sell their houses and buy another one). If the mortgage payment of a homeowner becomes greater than his/her income, he/she will be forced to sell the house and move out of the town that is represented in the model world. A number of homeowners will also put their houses on the market and leave the town each period (as defined in the "*ExitRate*" slider). On the other hand, the slider "*EntryRate*" defines the number of new prospective buyers arriving in the town. The graphic user interface slider "*MaxHomelessPeriod*" indicates how many ticks a potential buyer could be homeless (without a house) before giving up waiting for a house and leaving the town. It is important to note that since this model represents only home buyers, a homeless buyer agent could potentially be a renter planning to buy a home (i.e. a first-time home buyer), although renters are not specifically modeled.

New houses are built and put up for sale depending on the rate defined in the slider "*HouseConstructionRate*" and on the existence of vacant green patches (representing vacant land) in the grid that represents the town. Houses are assigned a quality index based on their location on the grid (depending on the quality index of adjacent houses). When a house is initially put for sale, the sale price assigned is based on the highest valuation offered by local realtors. The valuation is calculated based on the average selling price of houses in the realtor's territory. Houses that reach their end of life (defined by the slider "*HouseMeanLifetime*") or that lost too much of their value (based on the comparison of the median price of houses for sale and their sale price) are demolished.

As described, buyers can be new entrants to the market or those wishing to sell their house and move to another more expensive (or less expensive) house. In this specific model, new entrants to the market (who have no house to sell) will get priority when making an offer to buy a house. In addition, a buying chain concept is applied when offers are made: an offer will only be accepted if the house is already empty or if the seller of the house (the current owner occupier) succeeds in his/her offer to buy another property. No relative utility maximisation versus the house cost is assumed in this ABM. As explained by Gilbert, Hawksworth, and Swinney (2009), buyers will simply try to purchase the most expensive house they can afford. The purchase decision is very simple, and it does not consider any rational valuation or more complex analysis. The process to make an offer consists of searching for properties for sale that the buyer can afford that are not already under an offer. The offer will be made to the most expensive property found within the buyer's budget. The buyer's budget is the sum of the maximum value of the mortgage he/she can get in the new property, the projected sale price of the agent's current house and the available accumulated capital, subtracted by the amount he/she needs to pay back for the current mortgage and any duty's owed (in this model, for the English market, a stamp duty land tax was considered).

The repayment duration of the mortgage is defined by the "*MortgageDuration*" graphic user interface slider (set as default to 25 years). The maximum mortgage a buyer can get in a property being bought can be represented by the following formula, where "M" represents the maximum loan (mortgage), "W" represents the agent income and "i" represents the interest rate for the mortgage. The "GDS_{MAX}" ratio is defined in the "*Affordability*" slider:

$$M = W \times \frac{GDS_{MAX}}{i}$$

The maximum LTV is also considered in the process, as defined by the slider "*MaxLoanToValue*" in the user interface, so the buyers need to have sufficient funds for a deposit (that could be generated by the proceeds of the sale of the previous house subtracted by the balance of the previous mortgage or by savings accumulated during the years).

Realtors define the valuation of houses by looking at their sales records. A user interface slider defines the realtor's memory ("*RealtorsMemory*") in ticks, indicating for how long sales records will be considered in valuations. House valuations are calculated by the median price of houses for sale locally multiplied by the house's quality index and then multiplied by a realtor optimism factor (defined by a percentage on the "*RealtorOptimism*" slider). A normalisation factor is applied to prevent drastic changes in prices. If a house is not sold, its price will be reduced for the next tick continuously, until the house is sold or demolished. The price reduction is defined by the slider "*PriceDropRate*". This model has overlapping defined areas covered by different realtors. The sliders "*RealtorTerritory*" and "*Locality*" indicate the coverage area of each realtor.

It is possible to run the model and introduce shocks or new assumptions by modifying the sliders in the user interface or by directly changing the source code. Gilbert, Hawksworth, and Swinney (2009) achieved reproducing the main characteristics of a housing market with this model, for example the price elasticity to interest rates and the impact on housing prices created by the increase of new entrants (i.e., first time buyers) in the market. Since the model was created for the English housing market, it makes sense to review it and adjust some of its features to make it adequate to study the Canadian housing market. These updates and extensions will be described in the next section.

4.2 Updated Model for the Canadian Housing Market

Before starting to update and extend the model, the original code of the baseline model, proposed by Gilbert, Hawksworth, and Swinney (2009) was converted to run on version 6.2.2 of NetLogo [Wilensky (1999)], which is the latest stable version currently available. Once the model was running in version 6.2.2 of NetLogo, it was submitted to a series arbitrary changes to test the input controls in the graphic user interface and initially validate the functionality of the tool and the response to changes.

The user interface of the model was then updated, as shown *in Figure 8 - Updated User Interface of the Model for the Canadian Housing Market*. The controls were re-organized, and a larger space of the screen was setup for the model grid representing the world and the output series, which include the "*Median House Prices*" and the "*Median House Prices Variation*". Additional controls were created and some of the functionality which was not relevant for the Canadian housing market simulations was disabled. The functionality to export data from the output series (plotted charts) was tested and verified. NetLogo has built in functionality to export data from plots to comma-separated value (CSV) files. The charts information is exported in X and Y coordinates, where X represents the tick (the model time measure), and Y represents the variable being plotted to the output series in the chart. Exporting the data plotted to some of the output charts facilitates the validation of the model, which will be described later in the model validation section.

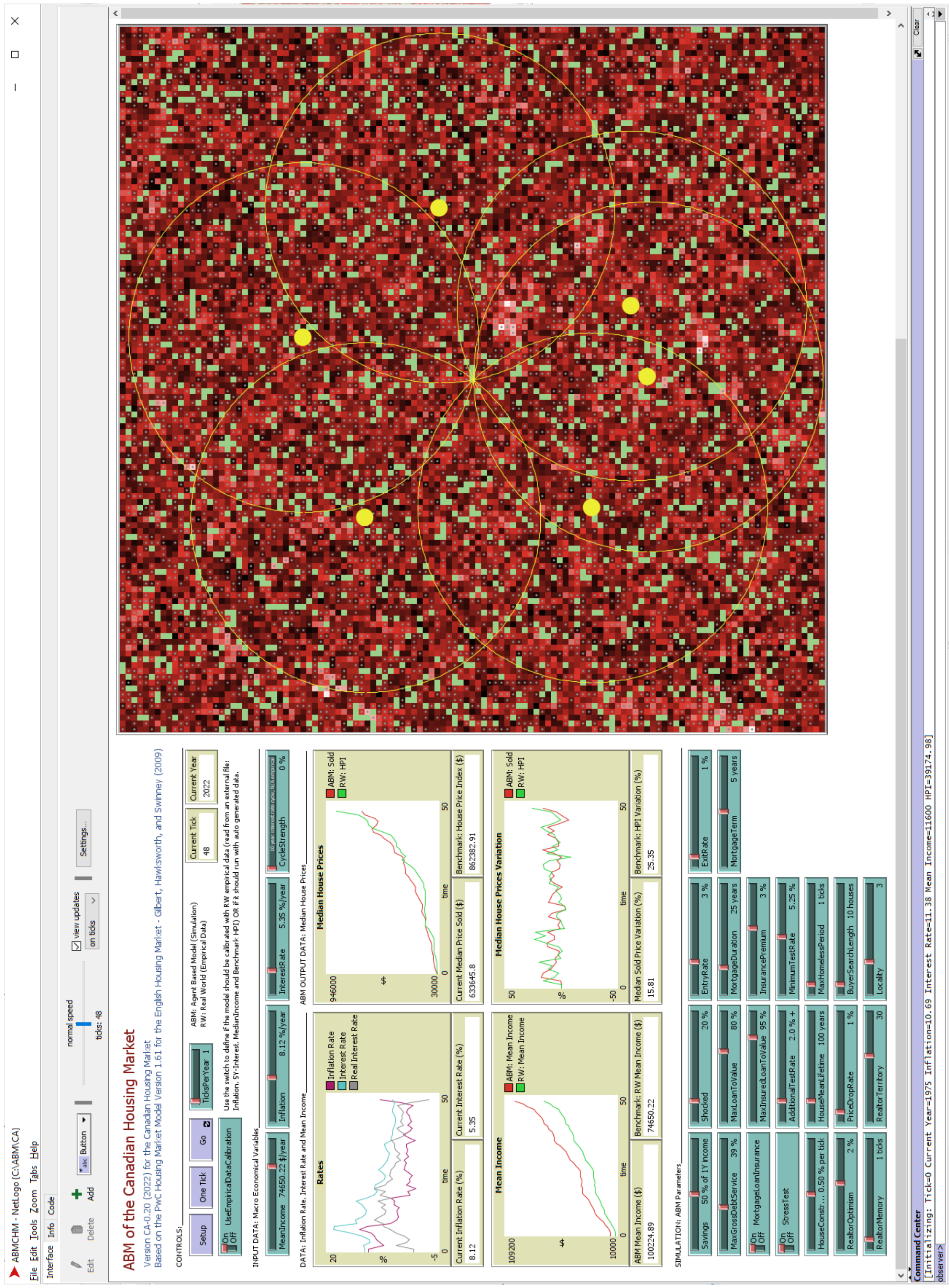

Figure 8 - Updated User Interface of the Model for the Canadian Housing Market

The next step was to identify the changes that needed to be done to the baseline model to update and extend it to be used for the Canadian housing market, so the model could be calibrated with empirical data from Canada to incorporate some key idiosyncrasies from this housing market. Since the original model does not read data from any external sources, all information related to variables and parameters needed to be entered via the user interface sliders and switches, which would make calibration with empirical data very time consuming.

NetLogo [Wilensky (1999)] has support to read inputs from text files, so new functionality was developed and incorporated to the baseline model to allow the model to read data from one external source in the form of a comma-separated value (CSV) text file. This allowed the model to be calibrated with empirical data in form of time series, representing the mortgage interest rates, inflation and household disposable income in Canada.

The original functionality, which allowed the user to generate these variables based on parameters entered directly in the user interface was maintained but disabled once the user enables the functionality "*UseEmpiricalDataCalibration*" through a newly integrated switch in the user interface, as demonstrated in *Figure 9 - ABM Controls and Variables*. The functionality to use empirical data calibration allows the user to read variables from the CSV text file, overriding the behaviour of the original model, so the new model runs with empirical data for the number of periods available in the input file, automatically updating the controls in the user interface. In addition to the mortgage interest rates, inflation and household disposable income, a time series with a calculated "real world" median house prices based on a house price index is also read but used exclusively as a benchmark to compare the output values generated by the ABM. The *Table 5 - Contents of a Sample Input CSV* File in the appendix shows the data read from the CSV text file.

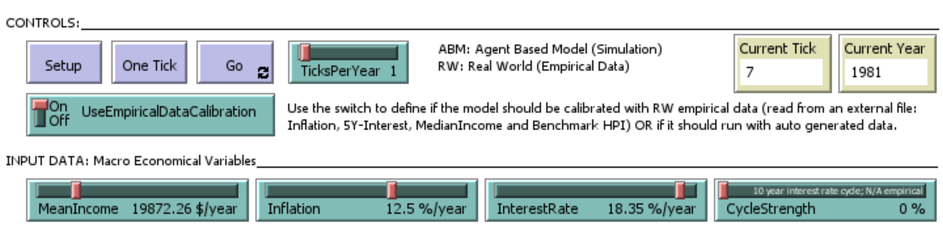

Figure 9 - ABM Controls and Variables

The frequency of periods in a year was also adjusted. The original model was running with 4 ticks per year (one per quarter of the year). It was observed, after running several simulations with different data periodicities, that simulations contained less noise and were more meaningful when running with 1 tick (or period) per year. As previously described in the data description chapter, 48 data points for each time series were used, from 1975 to 2022, which covers a relatively extensive period of time. In Canada, housing market activity generally follows an annual mini cycle, which roughly aligns to the seasons of the year. This is observable in the *Figure 10 - Variation of Residential Property Prices in Canada [BIS (2002)]*, generated from the *BIS Residential Property Price* database and retrieved from FRED [BIS (2022)]. The variation (percent change) of residential property prices in Canada have a cyclical component linked to the year quarters, generally starting to increase in the first quarter (Q1), from winter to spring, peaking in the second quarter (Q2), from spring to summer, starting to decrease in the third quarter (Q3), from summer to fall and lowering in fourth quarter (Q4), from fall to winter. This impacts quarterly readings even when comparing to the previous year. Reducing the frequency to once a year simplifies the model and reduces potential noise, as observed in the benchmark data from BIS (2022).

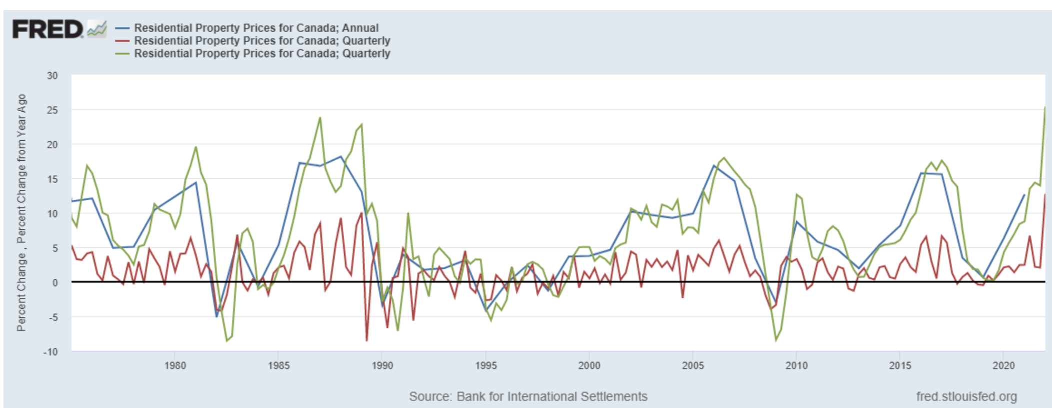

Figure 10 - Variation of Residential Property Prices in Canada [BIS (2002)]

The functionality to auto-generate an interest rate based on the values configured in the user interface (interest rate and 10-year cycle strength) was disabled when using empirical data. The 5-year mortgage interest rate data is read from the CSV file and the user interface is updated accordingly, with a plot of the rate and also the interest rate slider being automatically set to the rate that was read for that period. The mortgage interest rate is used in the ABM for mortgage payment calculations and also to determine the affordability of a property (affecting the decision to place an offer to buy a property), as it will be described in more details later.

The inflation and mean household income are also read from a CSV file, with the user interface being updated accordingly. The inflation rate is now plotted in a chart along with the mortgage interest rate and the difference of both series (an informative plot of the real interest rate), as demonstrated in *Figure 11 - Plot of Inflation and Interest Rate*.

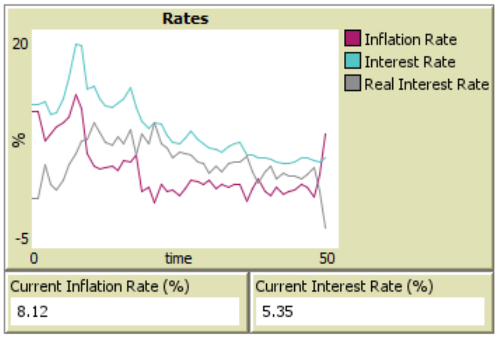

Figure 11 - Plot of Inflation and Interest Rate

The mean household income is plotted in a separated chart, displaying both the values read from the CSV file (real world data) and the one generated in the ABM (model data), as demonstrated *by Figure 12 - Plot of the ABM Mean Income and Real World Mean Income*.

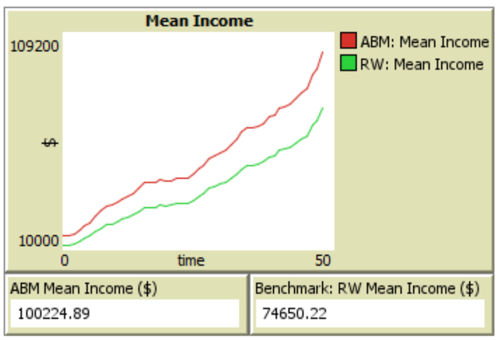

Figure 12 - Plot of the ABM Mean Income and Real World Mean Income

Similarly to the original version, every time a new home buyer agent is created, the model randomly assigns a mean income to the household using a gamma distribution and applying the values read for household mean income for the period. Once this is done and similarly to the original version of the model, a condition to avoid too low incomes (i.e. less than half of the mean income of all households) is applied. For this reason, the mean income of home buyers and homeowners in the ABM is higher than the real world observed income (read from the time series). This difference is expected, since this ABM only represents the property sales market, while the real-world data considers all households, including the households that do not participate in the property sales market.

In the original version of the model, the value defined for the existing households mean income was updated using the inflation rate in every period, as an income correction. This functionality was disabled when using empirical data, and the variation of the mean household income was used instead of the inflation rate since it is a more accurate rate to determine the income correction. Every period, a correction is applied to the existing agents incomes (W) using the household mean income variation read from the CSV file compared to the previous period (ΔW_{CSV}), as represented by the formula:

$$W_{tick} = W_{tick-1} \times (1 + \Delta W_{CSV})$$

The inflation rate read from the CSV file (I_{CSV}) is still used in the model, but to update the sale prices of houses (P) in every period, considering the inflation rate. This newly introduced price correction can be represented by the formula:

$$P_{tick} = P_{tick-1} \times (1 + I_{CSV})$$

The original functionality of the property valuation function was maintained. This function is used to determine the prices of new properties put for sale based on various parameters, including the realtor memory (in ticks). This parameter was adjusted to 1 tick for the Canadian housing market model. This means that the realtor agents will value properties by looking in its records for sales for the last period only. Since the model was adjusted to run with one tick (or period) per year, taking into consideration the record of sales from older periods did not produce more accurate valuations. As in the original version, the valuation is done every period for the houses put for sale, with the realtor agents considering their local records as baseline and then applying a quality index (assigned to the houses) to adjust the prices. A realtor optimism factor and a price drop rate, both configurable in the user interface as parameters are also taken into consideration. The realtor optimism increases the property value in every evaluation. Conversely, the prices of the houses that were not sold in a period are adjusted following the price drop rate.

Two important new developments for the Canadian version of the ABM were the introduction of mortgage terms and the possibility of having mortgage loan insurance to allow a smaller deposit (and consequently a larger loan-to-value ratio). As previously described, residential mortgages in Canada are usually amortised over 25 years and with terms varying from 6 months to 5 years. All calculations related to the mortgage were reviewed and some were updated.

Some functionality remained the same as in the original model. For example, the maximum mortgage an agent can contract to finance the purchase of a house is still determined by multiplying the agent income and the maximum gross-debt-service divided by the interest rate. The maximum GDS is configurable in the user interface. In the original version it was called "*Affordability*". The maximum GDS was set to 39%, as an approximation of the maximum GDS lenders usually require borrowers to qualify in Canada. For insured loans, this is also the ratio imposed by CMHC [CMHC (2022)]. In this ABM, the GDS is based on the mortgage payment (principal and interest). In the real world, in addition to the mortgage payments, it also considers the property taxes and heating, which are not in scope of this ABM.

The deposit or down-payment is still based on the savings an agent has accumulated over the years, as in the original model. If the agent buying a property already has a house to sell (moving from one house to another), it also includes the earnings from the sale of the first house, subtracted by the balance of the mortgage to settle once the transactions are completed.

The maximum loan-to-value is configurable in the user interface, and it was set as default to 80%, reflecting the maximum LTV percentage of uninsured loans in Canada. If the newly developed functionality of mortgage loan insurance is enabled (via a new "*MortgageLoanInsuranc*e" switch in the user interface), this value is overridden by the maximum insured loan to value, also configurable in a user interface slider. In this case, an insurance premium, configurable as a percentage of the loan, will be added to the mortgage, increasing the cost of the borrowing, but allowing the agent to access more financing.

Another new functionality introduced to the model for the Canadian housing market is the possibility of adding a stress test to the mortgage interest rate. This can be enabled via a new switch called "*StressTest*", along with two sliders: "*AdditionalTestRate*" and "*MininumTestRate*". When enabled, the mortgage rate during the house search process will be increased by the additional test rate, or the minimum test rate will be used (whichever is higher). This only impacts the process to identify houses the agent can buy. Mortgage repayments are still calculated with the mortgage interest rate, without the stress test.

The formula used to calculate the mortgage repayments (R) was reviewed and updated. The annual amount to be repaid is calculated based on the value of the mortgage (M), the interest rate (i) and the mortgage duration (D), as identified in the formula:

$$R = M \times i \times \frac{(1+i)^D}{((1+i)^D - 1)}$$

The mortgage duration (D) is the amortization period, configured in the "*MortgageDuration*" slider. It was set to 25 years for the Canadian housing market. A new configurable parameter named "*MortgageTerm*" was added in the user interface to define the mortgage term for renewal. It indicates the period a mortgage is negotiated at a determined rate. Once this period is completed, the mortgage will have to be renewed based on an updated interest rate at the renewal. This parameter is set to 5 years for the Canadian housing market. Every time a term is completed, the repayment will be recalculated using the mortgage repayments formula above, with an updated interest rate and the remaining balance of the mortgage. For mortgage term renewals, the mortgage age (elapsed years) will be subtracted from the mortgage duration.

Another change introduced in the model was the possibility to retrofit a house. Since houses not sold in a period have their value reduced following the price drop rate, it is possible that some houses become too undervalued. In the original model, these houses were demolished once their value drops to less than 10% of the median price of all houses sold, along with the houses that reached their end of life. In the new version, houses that reach their end of life are still demolished, but for houses that have at least 20 years of lifespan left, half of the savings of the homeowner will be used to retrofit the house. When a retrofit occurs, the homeowner savings is reduced by half and the house is re-evaluated by a realtor. The house is assigned a new updated evaluation, closer to the median price of houses sold. This operation represents homeowners investing in renovations to augment the property selling price.

The house end of life is defined for each house when it is created. It is based on the "*HouseMeanLifetime*" switch in the user interface. The model was updated to assign a lifetime to the house around the mean defined, with a random variation of 30 years (mean lifetime less 15 years to mean lifetime plus 15 years). The model includes other configurable parameters, as observable in the switches demonstrated in *Figure 13 - ABM Parameters*.

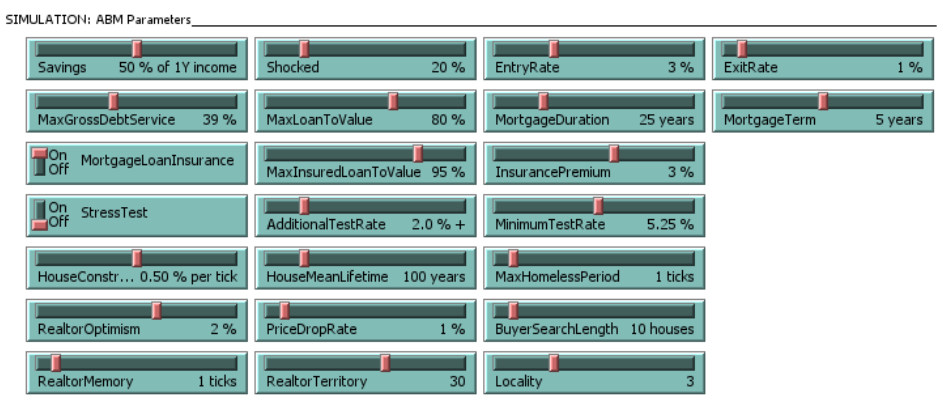

Figure 13 - ABM Parameters

The configurable parameters were attributed some baseline values, which will be used in the initial simulations and model validation. Some values were inherited from the base model and some values were adjusted to better reflect the Canadian housing market. The "*Savings*" switch was attributed a 50% value, indicating that owners will be assigned 50% of their income as savings when they are created. This is done only once when the owner agents are created.

The "*Shocked*" parameter indicates the percentage of agents that will receive an income shock (of +20% or -20% of their income) in a period. It was assigned the value of 20% of the total agents for the base simulations. The entry and exit rate parameters ("*EntryRate*" and "*ExitRate*") were assigned the values 3% and 1%, meaning that the (home buyers and sellers) agent population is increasing 2% every period in the model. It is important to note that participants of the rental market are not represented in this ABM, so this parameter does not represent the entire Canadian population growth. The base scenario was also set to run with a maximum GDS ("*MaxGrossDebtService*") of 39%, maximum LTV ("*MaxLoanToValue*") of 80%, mortgage amortization period ("*MortgageDuration*") of 25 years and terms ("*MortgageTerm*") of 5 years. A mortgage loan insurance ("*MortgageLoanInsurance*") increases the maximum LTV ("*MaxInsuredLoanToValue*") to 95%, with the increase of a 3% insurance premium ("*InsurancePremium*"). The stress test ("*StressTest*") functionality is disabled by default but can be configured to add a test rate to the mortgage interest rate ("*AdditionalTestRate*"), also considering a minimum test rate ("*MinumumTestRate*"). The base scenario has a house construction rate ("*HouseConstructionRate*") of 0,5% of houses per tick, with a lifespan ("*HouseMeanLifetime*") of 100 years. Agents without houses will keep looking for houses for 1 period ("*MaxHomelessPeriod*") before moving out of the town. Realtors have a 2% optimism ("*RealtorOptimism*") and prices of houses not sold are decreased by 1% ("*PriceDropRate*") if they are not sold in a period. The remaining parameters ("*BuyerSeachLengh*" of 10 houses, "*RealtorMemory*" of 1 period, "*RealtorTerritory*" of 30 patches and "*Locality*" of 3 patches) are used to adjust the valuation functionality performed by the realtors. This functionality and parameters were inherited from the original model but adjusted to run with a memory of 1 period, since the model was adjusted to run with one period per year, as previously explained.

The original model functionality to allow different geographies to be applied was disabled. This used to allow different types of clustering of houses in the grid, for example by income. Since the model is not being applied to a specific region of Canada, this functionality was not applicable. The functionality to consider a stamp duty tax was also disabled, since it does not apply for the Canadian housing market. Some of the plots in the original model were also hidden or disabled to streamline the model and have the user interface simplified.

In order to observe the model outputs and visually compare results with a benchmark, two new plots were introduced: one showing real-world median house prices (read as benchmark data only) and ABM median house prices (generated during the simulation) and one showing the variation of these median prices. The variation of median prices is calculated by subtracting the previous price from the current price, divided by the previous price. The newly introduced plots are shown in *Figure 14 - Plot of Median House Prices and Variation of Median House Prices*.

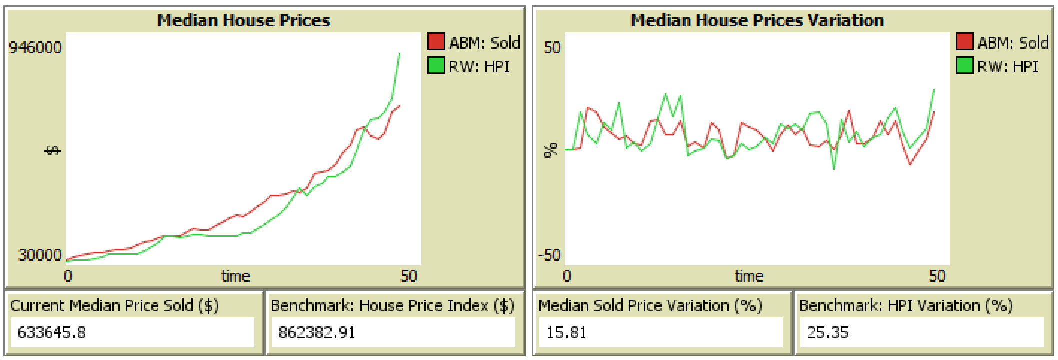

Figure 14 - Plot of Median House Prices and Variation of Median House Prices

The red lines in both plots (identified by "*ABM: Sold*") show the evolution of the median prices of houses sold in the model and its variation. The green lines (identified by "*RW: HPI*") show the benchmark values that were generated using the real-world house price index and its variation. These two plots of data are extremely useful not only to visually validate the data generated by the model and quickly compare it to the real-world data but also to export the data generated by the model for validation and analysis, as explained in the next section.

4.3 Validation of the Updated Model

The validation of an ABM is not a straightforward task since agents behaviours depend on other agents and the environment. Although it is possible to review each line of the algorithm that define how agents behave, another possibility is to validate the outcomes of the simulations.

For an initial validation of the updated model focused on the Canadian housing market, the outcomes of simulations can be compared to empirical data to validate plausibility. In this first step, the objective is to verify if the median housing prices and the variation on the median housing prices generated by the ABM follow the benchmark data. The correlation between the ABM results and the benchmark data was calculated. It is not expected that model estimations will follow the benchmark data perfectly, but it is expected that the generated estimations represent at some level the real-world data in the long term. In order to statistically validate this, a Johansen cointegration test was run in EViews (2022) to confirm that the non-stationary time series representing the median house prices generated by the ABM and the real-world benchmark are cointegrated, meaning that they will be in equilibrium in the long term.

Since the ABM generate results from the behaviours that emerge from the interactions between the individual agents, simulations will not produce identical results. Therefore, as a second step, a series of simulations was run, and the results were compared to validate consistency. Because of the time needed to manually run each simulation, the validation was done with 20 simulations. Every simulation takes on average 12 minutes to run, with an additional minute to complete the setup. The process is done sequentially for each simulation, which consists in clicking the "*Setup*" button, waiting for the setup to be completed, clicking the "*Go*" button to start the simulation, waiting for the simulation to be completed, and then exporting the data from both the "*Median House Prices*" and "*Median House Prices Variation*" charts that were generated by the model. This process was repeated 20 times, with identical values for the input variables and parameters. A total of 40 CSV files were exported from the model output plots (20 files of "*Median House Prices*" results and 20 files of "*Median House Prices Variation*" results). Each file contains 48 data points (estimations from 1975 to 2022). Data from the 20 simulations was combined and the mean (average) and standard deviation of values produced for each year was calculated. Although some variation is acceptable and anticipated, it is expected that the results of the simulations will converge. The correlation between the ABM results mean and the benchmark data was calculated. To complete the validation process, a Johansen cointegration test was repeated in EViews (2022), this time to validate that a time series with the mean (average) of all simulations is still cointegrated with time series of real-world data, indicating a long-term equilibrium, and implying the successful calibration of the model for the Canadian housing market.

To complete the model validation, a third step was performed to verify if the shocks that were introduced to the model, notably the interest rates increases (decreases) produced symmetric decreases (increases) in the property prices. This was done by comparing the data generated by one simulation of the ABM and the interest rate data inputs. The median house price calculated by the ABM was deflated using the CPI and then normalized (divided by the largest data point), so it could be compared at level to the mortgage interest rates used as input. The mortgage interest rates were similarly normalized, to allow the comparison to the ABM estimates. For this test, it is helpful to deflate the house price estimates, to remove the inflation component from prices. It is expected that the historical variation of interest rates affect the model estimations symmetrically. To confirm this, the correlation coefficient of the two normalized series was calculated to identify if there was a strong (or weak) and negative (or positive) correlation between the mortgage interest rate variable and the ABM median house price estimates. The correlation between the other variables (household income and inflation) was also verified similarly with the identification of the coefficient of correlation, but this time the median house prices generated by the ABM simulation was not deflated.

Once validations were completed, the model was run with modifications in some parameters (i.e. modification of the maximum loan to value and the introduction an additional mortgage stress test) and compared to a baseline scenario to confirm if changes would affect the median house prices generated by the model. For these simulations, one additional year (2023) was added to the input file with hypothetical values for the inflation (4.50%), average of the 5-year mortgage rate (7.35%) and household median income ($ 78,009.47).

The next chapter will present the results analysis, starting with results of the model validation and then presenting the results of some theoretical modifications to the loan-to-value and the addition of a stress test, compared to a baseline scenario generated by the model.

CHAPTER V

RESULTS ANALYSIS

In this chapter the estimations and results produced by the ABM will be presented and analysed, as described in the previous chapter.

5.1 Initial Validation of the Canadian Housing Market ABM

The median housing prices and the variation on the median housing prices generated by an ABM simulation were compared to the benchmark data, as shown in *Figure 15 - ABM and RW Data Compare: Residential Property Prices* and *Figure 16 - ABM and RW Data Compare: Residential Property Prices Variation.* It is possible to observe that both the housing prices and the variation on the housing prices generated by the ABM follow the benchmark data, although not perfectly.

It is observable in in *Figure 15 - ABM and RW Data Compare: Residential Property Prices* that the median house prices have grown consistently during the 48 years. The ABM and RW series have a strong positive correlation (0.9668). The series are non-stationary, and the chart suggests that they are cointegrated. To confirm the cointegration, a Johansen cointegration test was run assuming no deterministic trend in data and using the Schwarz information criteria to determine the lag length.

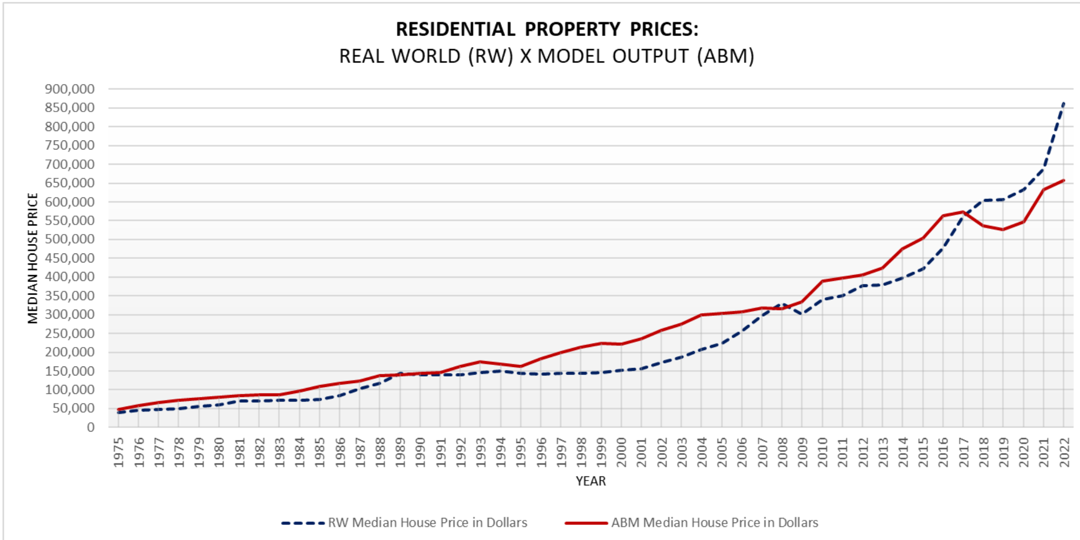

Figure 15 - ABM and RW Data Compare: Residential Property Prices

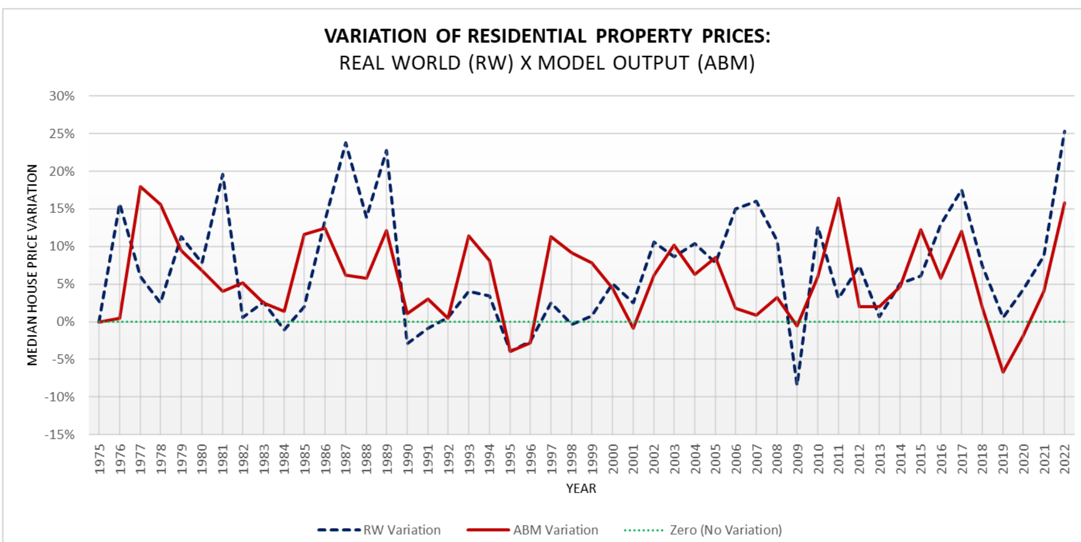

Figure 16 - ABM and RW Data Compare: Residential Property Prices Variation

The results of the cointegration test are shown in *Figure 17 - ABM and RW Median House Prices Cointegration Test*, indicating the residential property prices series generated by the model (ABM) and from real world (RW) empirical data have a long-run relationship.

Date: 10/09/22 Time: 00:09
Sample (adjusted): 1977 2022
Included observations: 46 after adjustments
Trend assumption: No deterministic trend
Series: ABM RW
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.569901	45.47792	12.32090	0.0000
At most 1 *	0.134901	6.665934	4.129906	0.0117

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.569901	38.81198	11.22480	0.0000
At most 1 *	0.134901	6.665934	4.129906	0.0117

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

ABM	RW
1.81E-06	-7.63E-06
2.26E-05	-2.19E-05

Unrestricted Adjustment Coefficients (alpha):

D(ABM)	-14383.47	2114.526
D(RW)	-7497.512	-6270.804

1 Cointegrating Equation(s): Log likelihood -1020.196

Normalized cointegrating coefficients (standard error in parentheses)

ABM	RW
1.000000	-4.209284
	(0.42795)

Adjustment coefficients (standard error in parentheses)

D(ABM)	-0.026068
	(0.00380)
D(RW)	-0.013588
	(0.00505)

Figure 17 - ABM and RW Median House Prices Cointegration Test

5.2 Validation of Simulation Convergence

Since every simulation generate results following a bottom-up process in which the median residential property prices will emerge from the individual agents behaviours, the resulting estimate of different simulations will not be identical. Despite variations, the results should converge, indicating that the model consistently generate satisfactory estimates. The convergence of 20 simulation results can be observed in *Figure 18 - ABM Simulations: Residential Property Prices Convergence* and *Figure 19 - ABM Simulations: Residential Property Prices Variation Convergence*.

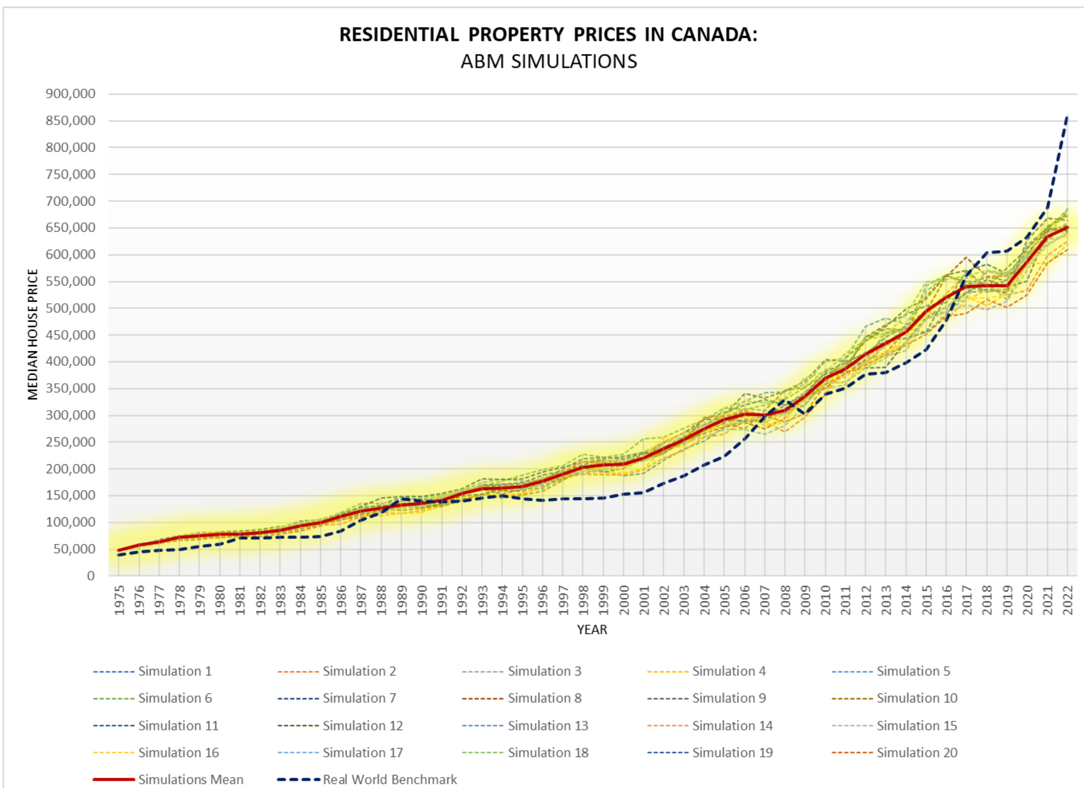

Figure 18 - ABM Simulations: Residential Property Prices Convergence

In *Figure 18 - ABM Simulations: Residential Property Prices Convergence* it is possible to observe that the dotted lines representing the median house prices estimated by each simulation converge. The mean (average) of simulations is represented by the middle red line.

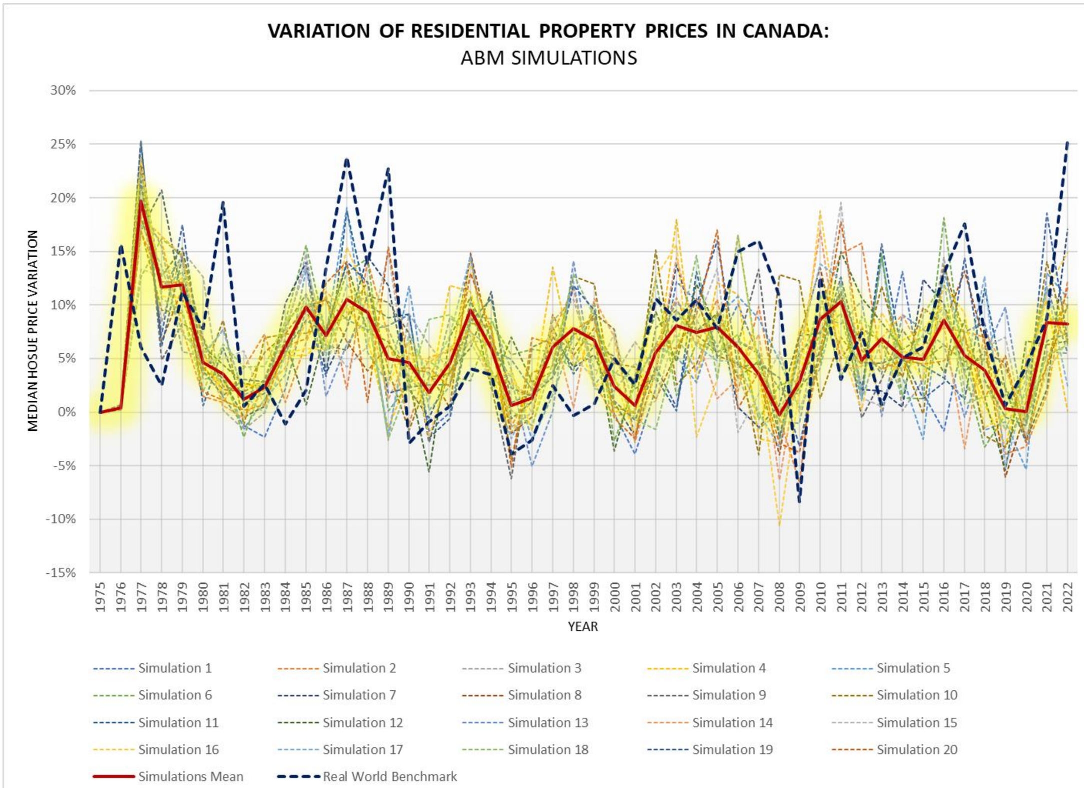

Figure 19 - ABM Simulations: Residential Property Prices Variation Convergence

The benchmark data and the mean of the ABM results are strongly correlated (0.9749). To confirm that the mean of the 20 simulations and the benchmark series are still cointegrated, a Johansen cointegration test was run similarly to the initial validation (assuming no deterministic trend in data and using and the Schwarz information criteria to determine the lag length), but this time using a series with the average of the median house prices estimated by the 20 simulations for each year. The results of the cointegration test are shown in *Figure 20 - ABM Simulations Average and RW Median House Prices Cointegration Test*. Results indicate that the average of estimations for residential property prices generated by 20 simulations of the model (ABM) and the real world (RW) data series are still cointegrated. This long-term equilibrium indicates the ABM calibration for the Canadian housing market is able to produce successive convergent estimates of residential property prices that are in long term equilibrium with the residential property prices empirical data used as benchmark.

Date: 10/09/22 Time: 00:44
Sample (adjusted): 1977 2022
Included observations: 46 after adjustments
Trend assumption: No deterministic trend
Series: ABM_AVG RW
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.572790	42.96157	12.32090	0.0000
At most 1	0.080080	3.839563	4.129906	0.0594

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.572790	39.12201	11.22480	0.0000
At most 1	0.080080	3.839563	4.129906	0.0594

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

ABM_AVG	RW
-3.51E-06	-4.32E-06
2.90E-05	-2.81E-05

Unrestricted Adjustment Coefficients (alpha):

D(ABM_AVG)	-7539.120	662.4916
D(RW)	-1192.516	-5995.151

1 Cointegrating Equation(s): Log likelihood -993.4857

Normalized cointegrating coefficients (standard error in parentheses)

ABM_AVG	RW
1.000000	1.230256
	(0.28774)

Adjustment coefficients (standard error in parentheses)

D(ABM_AVG)	0.026454
	(0.00370)
D(RW)	0.004184
	(0.01135)

Figure 20 - ABM Simulations Average and RW Median House Prices Cointegration Test

Further analysing the results, it is possible to observe *in Figure 19 - ABM Simulations: Residential Property Prices Variation Convergence* that the variation of median house prices differed between simulations from each year, but still converged, on average following the benchmark data in the long term. This can be explained by the fact that depending on the simulation, price variations may have occurred in the precedent or subsequent periods. Nevertheless, in the long run the resulting median prices were consistently convergent and cointegrated with the benchmark data, suggesting that the model is still (an imperfect) representation of the Canadian housing market.

Analysing the results of the 20 simulations in more detail, it was observed that the maximum standard deviation for the level series generated by the ABM (residential property prices) was 29,554 for the year 2016. The coefficient of variation of 5.67% was calculated by dividing this standard deviation by the mean value for that year (521,020.67). Further analysing the results, it was observed that the maximum coefficients of variation were 7.03% for the year 2008 (standard deviation of 21,746.18 and mean 309,381.10) and 6.82% for the year 2007 (standard deviation of 20,536.24 and mean 301,107.53). However, if each simulation is analysed individually, it is observable that these larger variations are corrected (or adjusted) in the subsequent periods, reinforcing the cointegration aspect of the ABM estimates for the median house prices and the real-world empirical data. The detailed results of the 20 simulations, included the calculated average and standard deviation for each year, are included in *Table 7 - ABM Simulations Results: Residential Property Prices (1/3)*, *Table 8 - ABM Simulations Results: Residential Property Prices (2/3)*, *Table 9 - ABM Simulations Results: Residential Property Prices (3/3)*, *Table 10 - ABM Simulations Results: Residential Property Prices Variation (1/3)*, *Table 11 - ABM Simulations Results: Residential Property Prices Variation (2/3)* and *Table 12 - ABM Simulations Results: Residential Property Prices Variation (3/3)* in the appendix.

The observed results also illustrate why in general, ABMs are not the best models to produce forecasts. The bottom-up simulation of behaviours in response to the environmental changes will produce equilibrium in the long term and are very useful in scenarios analysis. However, it is important to keep in mind that simulation estimates can vary in the short term.

5.3 Analysis of Interest Rate Correlation to ABM Results

The mortgage interest rate increases (decreases) produced symmetric decreases (increases) in the ABM property prices. This can be observed in *Figure 21 - Correlation of Interest Rates and ABM House Prices*, that shows the normalized interest rate and an ABM simulated median house prices, also normalized and deflated using the CPI. It is possible to observe that the distance between series widen and narrow, depending on the year (increases in interest rate in general bring the prices down, while decreases in interest rate in general bring prices up).

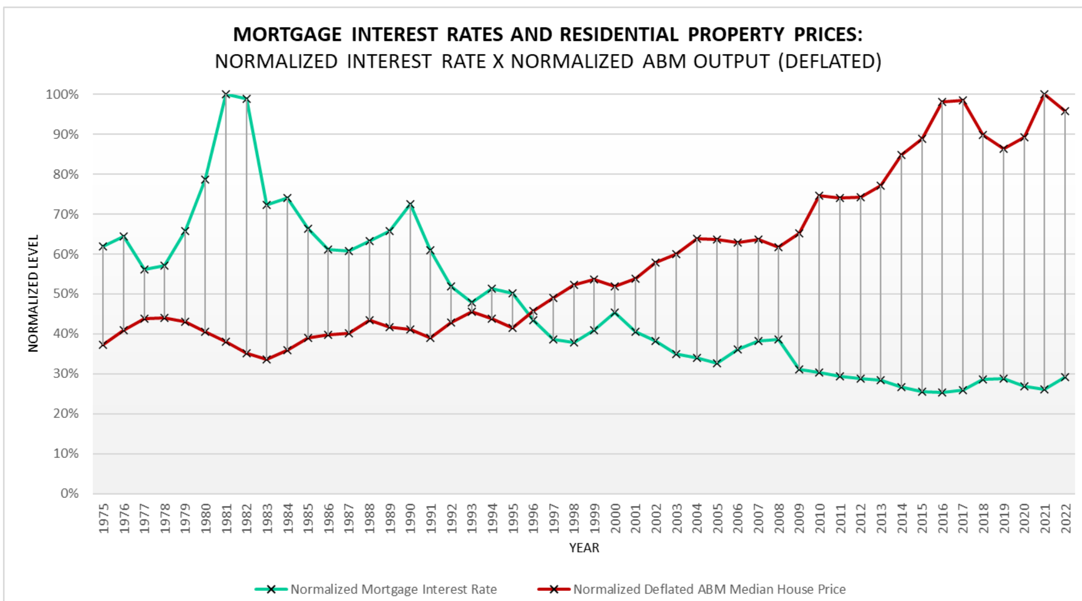

Figure 21 - Correlation of Interest Rates and ABM House Prices

The coefficient of correlation between the two series was calculated and resulted in a negative value closer to one than zero (-0.8235). This indicates a strong negative correlation between the two series, suggesting that the interest rate shocks symmetrically affect the model estimates. The *Table 13 - Correlation of Interest Rates and ABM House Prices* in the appendix contains the detailed values.

5.4 Analysis of Household Income and Inflation Correlation to ABM Results

The nominal household income and the inflation (represented by the CPI) have a positive correlation to the ABM property prices, as it can be observed in *Figure 22 - Correlation of Household Income, CPI and ABM House Prices*. For this analysis, the ABM median house prices were not deflated.

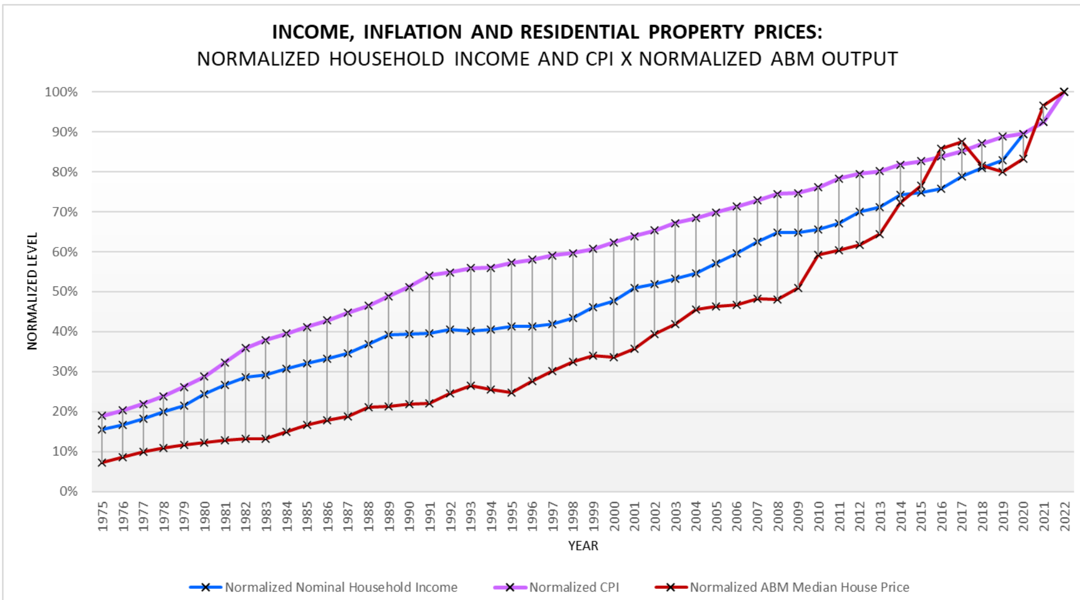

Figure 22 - Correlation of Household Income, CPI and ABM House Prices

The coefficient of correlation calculated between the normalized ABM median house price and the CPI indicates a strong positive correlation (0.9376). Likewise, the coefficient of correlation calculated between the normalized ABM median house price and the normalized nominal household income indicates an even stronger positive correlation (0.9807). *The Table 14 - Correlation of Household Income, Inflation and ABM House Prices* in the appendix includes the detailed calculated values. It is noteworthy to observe that in general the household incomes will accompany the inflation rate. The common notion that real estate prices would keep increasing for an indefinite period of time is actually an illustration of this correlation, which indicates nominal house prices in general follow the inflation (and consequently also the nominal household incomes).

5.5 Analysis of Changes in the ABM Parameters

A recent shock that affected the Canadian house market was caused by the COVID-19 pandemic. It is possible to note in the input data that 5-year mortgage rates were reduced in 2020 (if compared to previous years), at the same time that the median household disposable income increased with the introduction of various government support programs. This and other factors that are not modelled in this ABM (such the "*fear of missing out*" [Siddal (2019)] or the changes in preferences for more internal and external space caused by the pandemic) made house prices to grow beyond expectations. In the years that followed, the inflation, which was low in 2020, started to significantly increase. In order to control inflation, the central bank started increasing the policy interest rate, causing mortgage interest rates to increase.

The first proposed analysis is the alteration of the LTV (loan to value) ratio in 2020, which would reduce the leverage buyers can have to acquire homes from 2020 on, in some way offsetting the increase of the median household income. The *Figure 23 - Property Prices on Hypothetical LTV Adjustment Scenarios* shows the resulting median house prices estimated by the ABM for the baseline scenario and for two additional hypothetical scenarios, with the reduction of the LTV to from 95% to 93% and from 95% to 90%.

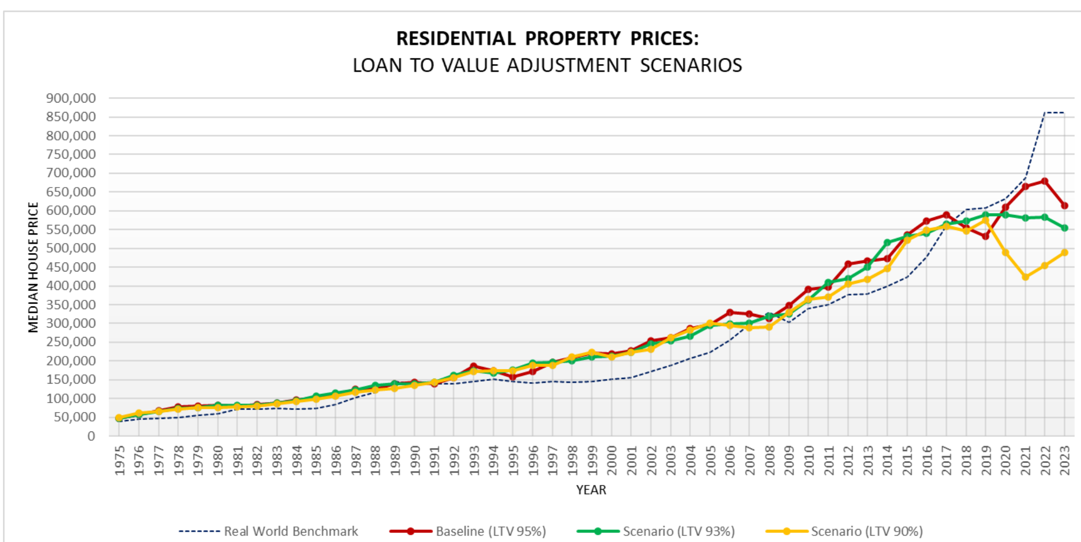

Figure 23 - Property Prices on Hypothetical LTV Adjustment Scenarios

It is possible to observe that reducing the LTV to 93% (which would mean requiring a minimum down payment of 7% instead of 5%) already reduced the increase in house prices generated by the model, in fact generating a moderate price decline. In the scenario where the LTV is reduced to 90% (requiring buyers to offer a minimum down payment of 10% instead of 5%), the model generated a correction in home prices, which lasted for 2 years (2020 - 2021). After that, prices started increasing, but from a lower level (2022 - 2023), as observable in *Figure 24 - Variation of Property Prices on Hypothetical LTV Adjustment Scenarios*.

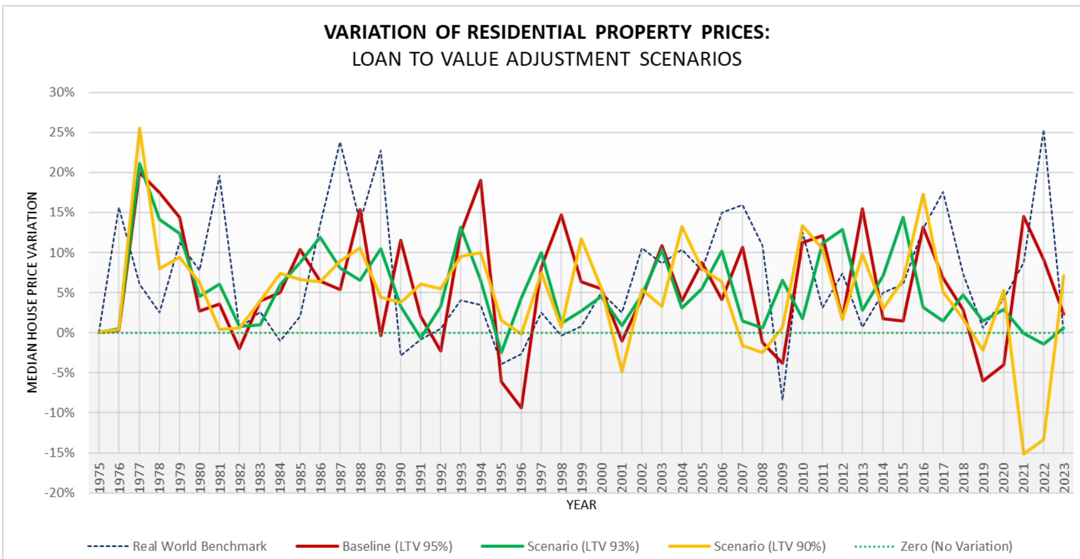

Figure 24 - Variation of Property Prices on Hypothetical LTV Adjustment Scenarios

It is important to keep in mind that this ABM does not consider all details of the housing market. It is an imperfect representation of the Canadian housing market based on various simplifications. The ABM is probably much more sensible to decreases in LTV than the real-world Canadian housing market. However, the results suggest that tightening the leverage conditions could have been a possible macroprudential tool for controlling excessive growth of house prices in a scenario where median household incomes have grown too fast in a short period (i.e. a median household disposable income increase of 7.1% in 2020 from the previous year [Statistics Canada (2020)]). In addition, this behaviour is consistent with the baseline model proposed by Gilbert, Hawksworth, and Swinney (2009).

The second proposed analysis is the addition of a supplementary interest rate constraint in the form of a stress test, forcing the buyers to qualify at a rate higher than the one used in the model. It is important to note that the ABM was originally calibrated without considering the current stress test (the already in place OSFI Guidelines B-20 and B-21 [OSFI (2017)]). This was due to the simplification of assuming all mortgages are based on the 5-year interest rate. In the real-world, a considerable number of mortgages are contracted with variable rates (which may be significantly lower than the ones used as input in the ABM during some periods). There are also other lending arrangements not considered in the model (e.g. mortgages that would also consider the buyer current home equity). Applying a rate increase test in the top of the 5-year mortgage interest rate has an exacerbated effect in the ABM (which is constrained to the 5-year rate, usually higher than variable rates for example), if compared to the real-world. This is, however, a good exercise to observe the effect of theoretical interest rate increases.

It is possible to observe in *Figure 25 - Property Prices on Hypothetical Stress Test Adjustment Scenarios* the effect of an additional stress test of 1% and of 2% for buyers trying to qualify for a mortgage.

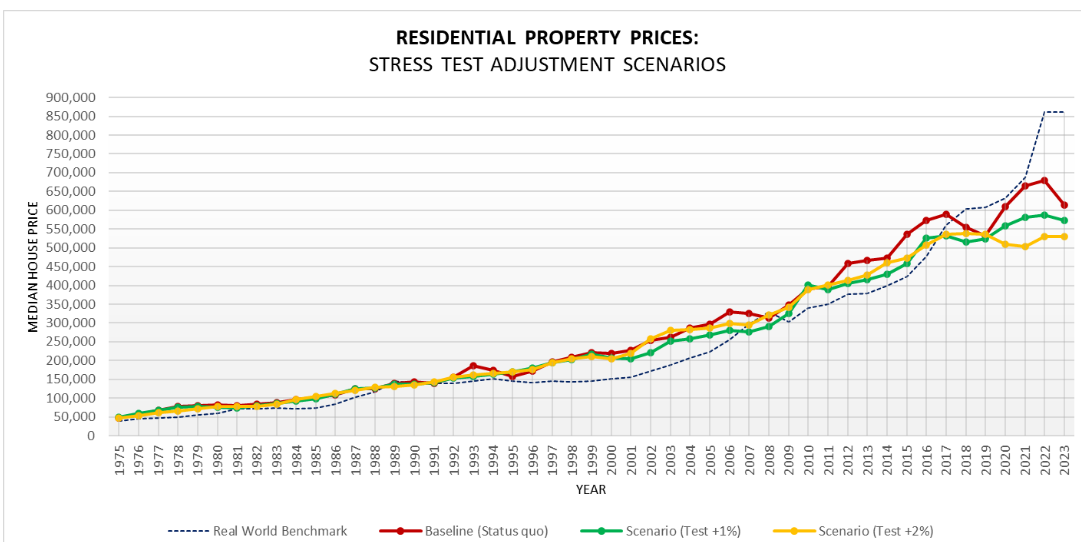

Figure 25 - Property Prices on Hypothetical Stress Test Adjustment Scenarios

It is observable that requiring borrowers to qualify at an additional 1% already affected house prices, which followed a more stable path going forward, preventing abrupter price decreases. The scenario where an additional 2% rate was tested presented a slight correction (2020 - 2021), followed by price increases in the subsequent years (2022 - 2023), as observed in *Figure 26 - Variation of Property Prices on Hypothetical Stress Test Adjustment Scenarios*.

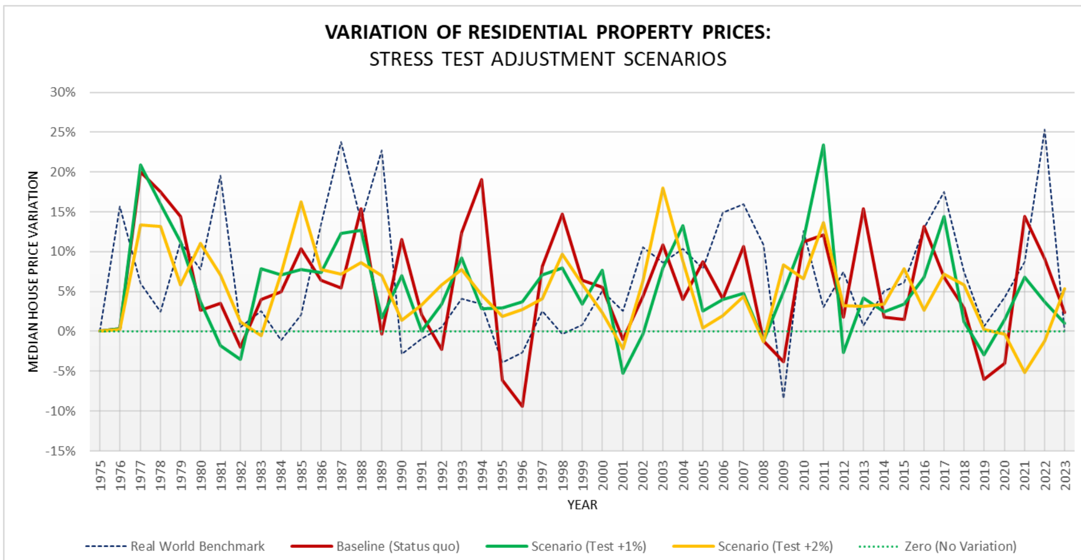

Figure 26 - Variation of Property Prices on Hypothetical Stress Test Adjustment Scenarios

In both cases (constrained LTV and additional stress test), the ABM compensated for the price adjustments after some time, but prices remained lower than in the baseline scenario. It is very important to note that the ABM is not supposed to be used to forecast changes in the housing market or the exact level of change a macroprudential policy would cause. The model is probably much more sensible to some changes than the real-world economy. However, it demonstrates in its small world representation of the real-world that both the stress test and the reduction of leverage are very effective policies to constrain exacerbated growth in housing prices. These policies could theoretically focus only on the housing market to compensate the increase in median household disposable income or decrease in interest rates when other macroeconomic factors beyond the housing market need to be considered, as it happened during the shock cased by the start of the COVID-19 pandemic.

CONCLUSION

Studying the Canadian housing market is key to understand the Canadian economy. The real estate sector represents a large portion of many Canadians wealth, and it was greatly affected by the recent shock caused by the start of the COVID-19 pandemic. Mortgage interest rates and household disposable income were impacted by government intervention, which affected not only the general inflation rate, but also the inflation of housing prices.

Economic models allow a simplified representation of complex economies, so they can be studied, allowing complex problems to be analysed. Traditionally, the Dynamic Stochastic General Equilibrium (DSGE) models have been successfully used to model economic subjects, including the housing market, generally with the assumption of rationality and homogeneity. In order to complement these models, computational models, including Agent-Based Models (ABMs) have been developed. These models employ computational power to build bottom-up systems where the successive behaviours in response to the environment of autonomous and heterogeneous agents allow the emergence of an aggregate collective behaviour. Although each agent's decisions and behaviours are unsophisticated if analysed in isolation, the collective behaviour that emerges from all agents is fascinating and revealing, in an analogy to what can be observed in beehives or ant colonies, for example.

This work proposed an ABM for the Canadian housing market, based on the work developed by Gilbert, Hawksworth, and Swinney (2009) for the English housing market and using the NetLogo [Wilensky (1999)] environment to model the Canadian housing market. The original model was extended and adapted to reflect the Canadian housing market (in a simplified way) and modified to run using empirical data from the Canadian economy as input for mortgage interest rate, household disposable income and inflation rate. Some modifications were inspired by other ABMs developed for different housing markets.

The ABM developed for the Canadian housing market successfully produced consecutive convergent estimates of residential property prices strongly correlated to the residential property prices empirical data from the Canadian housing market. Although prices increases and decreases varied in the short term, the median prices generated by the ABM were consistently convergent and cointegrated with the real-world benchmark data in the long run. This characterizes well one of the ABM’s known challenges since they are not well-suited to produce short term forecasts. On the other hand, ABMs are very useful for scenario analysis and to inform macroprudential policy. The creation of bottom-up simulation of behaviours in response to the environmental changes allow the simulation of potential policy intervention, for example leverage limitation or addition of a mortgage rate stress test.

Results suggest that mortgage interest rates are negatively correlated to house prices, while household incomes and inflation are strongly positively correlated to house prices. In general, nominal house prices followed the inflation and nominal household incomes, which largely accompany the inflation rate. This illustrates the sentiment that real estate prices would keep increasing indefinitely, especially when the nominal housing prices are analysed. Mortgage interest rate increases produced price declines. This is also observed with the addition of mortgage rate stress tests, which generated moderate price declines, also preventing abrupter price variations. In the ABM, households will spend as much as they can access to afford a house. Constraining loan-to-value levels effectively reduced leverage and consequently house prices. The ABM allows the simulation of potential outcomes of these and other macroprudential policies that could be focused specifically on the housing market. Possible future enhancements include the consideration of the real estate investment sector (rental properties), inclusion of dwelling types (detached houses, attached houses and multi-units) and use of more granular input data (i.e. consideration of variable and fixed interest rates).

Access to adequate and affordable housing is not only a human right but also a key foundation for households and societies to prosper. Appropriate and informed macroprudential regulation is key to manage housing sector risks. The further development of good models and the increasing availability of data will allow additional enhancements in risk management to promote financial stability and a stronger Canadian economy.

APPENDIX

The tables in the appendix present the data used in this research. The sources are briefly described below, along with the description of calculations and adjustments that were made to the data so it could be used in the agent-based model.

Mortgage interest rates are presented in *Table 1 - Yearly Average of 5-Year Conventional Mortgage Rates in Canada*. Data was obtained from Bank of Canada (2022) website, weekly series V80691335 (*5-year conventional mortgage*). The simple average of the weekly rates by year was calculated to obtain the yearly average from 1975 to 2022.

Inflation rates are presented in *Table 2 - Yearly Average of Inflation Rate in Canada*. Data from 1975 to 2021 was obtained from Statistics Canada (2022) website, table 18-10-0005-01 (*Consumer Price Index, annual average, not seasonally adjusted*). For the current year (2022), the CPI data was inputted based on the latest data available (July 2022) from Statistics Canada website, table 18-10-0004-01 (*Consumer Price Index, monthly, not seasonally adjusted*). The variation (percentual annual difference of the CPI) was calculated from 1975 to 2022 by comparing the current year CPI to the previous year CPI.

Household disposable income is presented in *Table 3 - Yearly Median Household Disposable Income in Canada*. Data for the median after-tax income of economic families and persons not in an economic family from 1976 to 2020 was obtained from Statistics Canada (2022) website, table 11-10-0190-01 (*Market income, government transfers, total income, income tax and after-tax income by economic family type*). Data was inputted for 1975, 2021 and 2022 based on the nearest data point available (i.e. the 1976 median household income in 2020 dollars was applied to 1975, and the 2020 median household income in 2020 dollars was applied to 2021 and 2022) and subsequently converted to a nominal income using the CPI.

Since the original series have the median household income converted to 2020 dollars, the CPI with reference in 2002 (also presented in *Table 2 - Yearly Average of Inflation Rate in Canada*), obtained from Statistics Canada (2022) website was re-based to 2020 and then used to calculate the nominal median household income from 1975 to 2022. This is an important conversion for this model since the simulation uses the nominal median household incomes for each year.

Residential property prices in Canada are presented in *Table 4 - Residential Property Prices in Canada*. Data was obtained from the Federal Reserve Bank of St. Louis (FRED) website, which includes series from the Bank for International Settlements (BIS) *Residential Property Price* database. Both the series QCAN628BIS (*Residential Property Prices for Canada*) and QCAR628BIS (*Real Residential Property Prices for Canada*), from the Bank for International Settlements, were retrieved from FRED. The second series (real property prices) is deflated using the CPI. The variation (percentual annual difference of the house price index in the series) was calculated from 1975 to 2022 by comparing the current year first quarter HPI to the previous year first quarter HPI. A median house price in dollars was calculated by re-basing both HPI series to 2021 (originally in 2010) and than multiplying the index by the *MLS® Average Prices* in 2021, obtained from CMHC (2022) web site. With this, a series of nominal median house price in dollars was obtained, along with a series of deflated median house price in dollars with reference in 2021. The nominal median house price series and the variation of the original HPI is used solely to validate the model, as this data was used only for benchmarking and validation purposes.

A sample of the data inputted to the model is presented in *Table 5 - Contents of a Sample Input CSV File*. A sample of the data of output files is presented in *Table 6 - Contents of Sample Output CSV Files*. The detailed results of 20 successive ABM simulations is presented in *Table 7, Table 8 and Table 9 - ABM Simulations Results: Residential Property Prices* and *Table 10, Table 11 and Table 12 - ABM Simulations Results: Residential Property Prices Variation (3/3).* Variables correlation analysis is shown in *Table 13 - Correlation of Interest Rates and ABM House Prices* and *Table 14 - Correlation of Household Income, Inflation and ABM House Prices.*

Table 1 - Yearly Average of 5-Year Conventional Mortgage Rates in Canada

Year	Average	Std. Dev.	Maximum	Minimum	Observations
1975	**0.1138**	0.0056	0.1200	0.1050	53
1976	**0.1182**	0.0027	0.1200	0.1075	52
1977	**0.1030**	0.0015	0.1075	0.1025	52
1978	**0.1049**	0.0040	0.1125	0.1025	52
1979	**0.1205**	0.0125	0.1475	0.1100	52
1980	**0.1445**	0.0125	0.1750	0.1300	53
1981	**0.1835**	0.0217	0.2175	0.1550	52
1982	**0.1815**	0.0179	0.1975	0.1450	52
1983	**0.1328**	0.0045	0.1475	0.1250	52
1984	**0.1360**	0.0085	0.1525	0.1250	52
1985	**0.1217**	0.0057	0.1350	0.1150	52
1986	**0.1121**	0.0038	0.1200	0.1050	53
1987	**0.1116**	0.0058	0.1225	0.1000	52
1988	**0.1162**	0.0039	0.1225	0.1100	52
1989	**0.1206**	0.0033	0.1275	0.1175	52
1990	**0.1332**	0.0076	0.1425	0.1200	52
1991	**0.1120**	0.0066	0.1250	0.0990	52
1992	**0.0954**	0.0058	0.1050	0.0850	53
1993	**0.0878**	0.0051	0.0950	0.0775	52
1994	**0.0941**	0.0116	0.1075	0.0725	52
1995	**0.0920**	0.0074	0.1075	0.0845	52
1996	**0.0798**	0.0055	0.0850	0.0695	52
1997	**0.0708**	0.0029	0.0765	0.0670	53
1998	**0.0694**	0.0020	0.0755	0.0660	52
1999	**0.0752**	0.0053	0.0825	0.0660	52
2000	**0.0834**	0.0017	0.0875	0.0795	52
2001	**0.0743**	0.0037	0.0795	0.0645	52
2002	**0.0700**	0.0027	0.0745	0.0670	52
2003	**0.0642**	0.0026	0.0685	0.0580	53
2004	**0.0625**	0.0028	0.0670	0.0570	52
2005	**0.0598**	0.0018	0.0630	0.0570	52
2006	**0.0664**	0.0020	0.0695	0.0630	52
2007	**0.0701**	0.0036	0.0754	0.0644	52
2008	**0.0710**	0.0023	0.0754	0.0665	53
2009	**0.0570**	0.0029	0.0675	0.0525	52
2010	**0.0557**	0.0032	0.0625	0.0519	52
2011	**0.0539**	0.0015	0.0569	0.0519	52
2012	**0.0527**	0.0008	0.0544	0.0514	52
2013	**0.0523**	0.0009	0.0534	0.0514	52
2014	**0.0489**	0.0019	0.0534	0.0479	53
2015	**0.0467**	0.0006	0.0479	0.0464	52
2016	**0.0466**	0.0004	0.0474	0.0464	52
2017	**0.0476**	0.0014	0.0499	0.0464	52
2018	**0.0527**	0.0011	0.0534	0.0499	52
2019	**0.0527**	0.0008	0.0534	0.0519	52
2020	**0.0495**	0.0015	0.0519	0.0479	53
2021	**0.0479**	0.0000	0.0479	0.0479	52
2022	**0.0535**	0.0060	0.0614	0.0479	*(*) 37*

(*) For the current year (2022), only 37 observations are available.
Source: Bank of Canada (2022) *and author's calculations.*

Table 2 - Yearly Average of Inflation Rate in Canada

Year	CPI (All Items)	CPI Variation	CPI Variation (%)
1975	29.00	**0.1069**	10.69%
1976	31.10	**0.0724**	7.24%
1977	33.60	**0.0804**	8.04%
1978	36.60	**0.0893**	8.93%
1979	40.00	**0.0929**	9.29%
1980	44.00	**0.1000**	10.00%
1981	49.50	**0.1250**	12.50%
1982	54.90	**0.1091**	10.91%
1983	58.10	**0.0583**	5.83%
1984	60.60	**0.0430**	4.30%
1985	63.00	**0.0396**	3.96%
1986	65.60	**0.0413**	4.13%
1987	68.50	**0.0442**	4.42%
1988	71.20	**0.0394**	3.94%
1989	74.80	**0.0506**	5.06%
1990	78.40	**0.0481**	4.81%
1991	82.80	**0.0561**	5.61%
1992	84.00	**0.0145**	1.45%
1993	85.60	**0.0190**	1.90%
1994	85.70	**0.0012**	0.12%
1995	87.60	**0.0222**	2.22%
1996	88.90	**0.0148**	1.48%
1997	90.40	**0.0169**	1.69%
1998	91.30	**0.0100**	1.00%
1999	92.90	**0.0175**	1.75%
2000	95.40	**0.0269**	2.69%
2001	97.80	**0.0252**	2.52%
2002	***100.00***	**0.0225**	2.25%
2003	102.80	**0.0280**	2.80%
2004	104.70	**0.0185**	1.85%
2005	107.00	**0.0220**	2.20%
2006	109.10	**0.0196**	1.96%
2007	111.50	**0.0220**	2.20%
2008	114.10	**0.0233**	2.33%
2009	114.40	**0.0026**	0.26%
2010	116.50	**0.0184**	1.84%
2011	119.90	**0.0292**	2.92%
2012	121.70	**0.0150**	1.50%
2013	122.80	**0.0090**	0.90%
2014	125.20	**0.0195**	1.95%
2015	126.60	**0.0112**	1.12%
2016	128.40	**0.0142**	1.42%
2017	130.40	**0.0156**	1.56%
2018	133.40	**0.0230**	2.30%
2019	136.00	**0.0195**	1.95%
2020	137.00	**0.0074**	0.74%
2021	141.60	**0.0336**	3.36%
2022	*(*) 153.10*	**0.0812**	8.12%

(*) The CPI (All Items) 2022 data was inputted based on July 2022 CPI [Statistics Canada (2022)] since the current year annual data is not yet available. Variation was calculated based on the index for the previous year (2021). Source: Statistics Canada (2022) and author's calculations.

Table 3 - Yearly Median Household Disposable Income in Canada

Year	Median Household Income in 2020 Dollars	CPI Ref. 2002	CPI Ref. 2020	Nominal Median Household Income
1975	*(*) 54,800.00*	29.00	21.17	**11,600.00**
1976	54,800.00	31.10	22.70	**12,440.00**
1977	55,700.00	33.60	24.53	**13,660.73**
1978	55,800.00	36.60	26.72	**14,907.15**
1979	55,100.00	40.00	29.20	**16,087.59**
1980	56,500.00	44.00	32.12	**18,145.99**
1981	55,000.00	49.50	36.13	**19,872.26**
1982	53,300.00	54.90	40.07	**21,358.91**
1983	51,400.00	58.10	42.41	**21,798.10**
1984	51,900.00	60.60	44.23	**22,957.23**
1985	52,100.00	63.00	45.99	**23,958.39**
1986	51,800.00	65.60	47.88	**24,803.50**
1987	51,600.00	68.50	50.00	**25,800.00**
1988	52,900.00	71.20	51.97	**27,492.55**
1989	53,700.00	74.80	54.60	**29,319.42**
1990	51,500.00	78.40	57.23	**29,471.53**
1991	48,800.00	82.80	60.44	**29,493.72**
1992	49,300.00	84.00	61.31	**30,227.74**
1993	48,000.00	85.60	62.48	**29,991.24**
1994	48,300.00	85.70	62.55	**30,213.94**
1995	48,300.00	87.60	63.94	**30,883.80**
1996	47,500.00	88.90	64.89	**30,822.99**
1997	47,300.00	90.40	65.99	**31,211.09**
1998	48,600.00	91.30	66.64	**32,388.18**
1999	50,800.00	92.90	67.81	**34,447.59**
2000	51,100.00	95.40	69.64	**35,583.50**
2001	53,200.00	97.80	71.39	**37,977.81**
2002	53,100.00	***100.00***	72.99	**38,759.12**
2003	53,000.00	102.80	75.04	**39,769.34**
2004	53,400.00	104.70	76.42	**40,810.07**
2005	54,600.00	107.00	78.10	**42,643.80**
2006	55,800.00	109.10	79.64	**44,436.35**
2007	57,300.00	111.50	81.39	**46,634.67**
2008	58,100.00	114.10	83.28	**48,388.39**
2009	58,000.00	114.40	83.50	**48,432.12**
2010	57,500.00	116.50	85.04	**48,895.99**
2011	57,300.00	119.90	87.52	**50,147.96**
2012	58,800.00	121.70	88.83	**52,233.28**
2013	59,200.00	122.80	89.64	**53,063.94**
2014	60,600.00	125.20	91.39	**55,380.44**
2015	60,400.00	126.60	92.41	**55,814.89**
2016	60,400.00	128.40	93.72	**56,608.47**
2017	61,900.00	130.40	95.18	**58,917.96**
2018	62,100.00	133.40	97.37	**60,468.18**
2019	62,400.00	136.00	99.27	**61,944.53**
2020	66,800.00	137.00	***100.00***	**66,800.00**
2021	*(*) 66,800.00*	141.60	103.36	**69,042.92**
2022	*(*) 66,800.00*	153.10	111.75	**74,650.22**

(*) Data was inputted for 1975, 2021 and 2022 based on the nearest data point available, subsequently adjusted by the CPI (originally referenced in 2002 and converted to 2020) to generate a nominal median household income. Source: Statistics Canada (2022) *and author's calculations.*

Table 4 - Residential Property Prices in Canada

	Nominal House Price Index				***Real: Deflated House Price Index (Ref. 2021)***			
Year	**HPI-2010**	**HPI-2021**	**HPI Var.**	**(*) House $**	**HPI-2010**	**HPI-2021**	**HPI Var.**	**(*) House $**
1975	11.4381	5.6941	**9.26%**	**39,174.98**	47.8619	28.4160	**-2.12%**	**195,499.50**
1976	13.2321	6.5872	**15.68%**	**45,319.35**	50.5830	30.0316	**5.69%**	**206,614.26**
1977	14.0312	6.9850	**6.04%**	**48,056.24**	50.3338	29.8836	**-0.49%**	**205,596.37**
1978	14.3809	7.1591	**2.49%**	**49,253.94**	47.3133	28.0903	**-6.00%**	**193,258.66**
1979	16.0000	7.9651	**11.26%**	**54,799.29**	48.3181	28.6869	**2.12%**	**197,362.92**
1980	17.2446	8.5847	**7.78%**	**59,061.99**	47.5552	28.2339	**-1.58%**	**194,246.74**
1981	20.6202	10.2652	**19.57%**	**70,623.27**	50.6304	30.0597	**6.47%**	**206,807.88**
1982	20.7326	10.3211	**0.55%**	**71,008.23**	45.7033	27.1345	**-9.73%**	**186,682.36**
1983	21.2695	10.5884	**2.59%**	**72,847.09**	43.5359	25.8477	**-4.74%**	**177,829.27**
1984	21.0406	10.4744	**-1.08%**	**72,063.12**	40.9559	24.3159	**-5.93%**	**167,290.85**
1985	21.4693	10.6879	**2.04%**	**73,531.40**	40.3300	23.9443	**-1.53%**	**164,734.26**
1986	24.3538	12.1238	**13.44%**	**83,410.68**	43.8620	26.0413	**8.76%**	**179,161.28**
1987	30.1561	15.0123	**23.83%**	**103,283.30**	52.2128	30.9992	**19.04%**	**213,271.44**
1988	34.3351	17.0927	**13.86%**	**117,596.19**	57.0999	33.9007	**9.36%**	**233,233.57**
1989	42.1436	20.9799	**22.74%**	**144,339.96**	67.0842	39.8285	**17.49%**	**274,016.02**
1990	40.9282	20.3749	**-2.88%**	**140,177.27**	61.7992	36.6907	**-7.88%**	**252,428.61**
1991	40.5703	20.1967	**-0.87%**	**138,951.47**	57.5528	34.1696	**-6.87%**	**235,083.51**
1992	40.7909	20.3065	**0.54%**	**139,707.02**	56.9638	33.8199	**-1.02%**	**232,677.65**
1993	42.4600	21.1375	**4.09%**	**145,423.61**	58.0646	34.4735	**1.93%**	**237,174.04**
1994	43.9251	21.8668	**3.45%**	**150,441.51**	59.7409	35.4687	**2.89%**	**244,021.15**
1995	42.2144	21.0152	**-3.89%**	**144,582.44**	56.5556	33.5776	**-5.33%**	**231,010.29**
1996	41.0989	20.4599	**-2.64%**	**140,761.91**	54.2704	32.2208	**-4.04%**	**221,676.03**
1997	42.1311	20.9737	**2.51%**	**144,297.14**	54.4805	32.3456	**0.39%**	**222,534.22**
1998	41.9896	20.9033	**-0.34%**	**143,812.51**	53.7406	31.9063	**-1.36%**	**219,511.98**
1999	42.3143	21.0649	**0.77%**	**144,924.60**	53.7427	31.9075	**0.00%**	**219,520.56**
2000	44.4371	22.1217	**5.02%**	**152,195.09**	54.9799	32.6421	**2.30%**	**224,574.10**
2001	45.5567	22.6791	**2.52%**	**156,029.67**	54.8502	32.5651	**-0.24%**	**224,044.32**
2002	50.3892	25.0848	**10.61%**	**172,580.77**	59.7421	35.4694	**8.92%**	**244,026.06**
2003	54.7305	27.2460	**8.62%**	**187,449.53**	62.1073	36.8737	**3.96%**	**253,687.09**
2004	60.4329	30.0847	**10.42%**	**206,980.00**	67.9603	40.3486	**9.42%**	**277,594.59**
2005	65.1636	32.4398	**7.83%**	**223,182.43**	71.7559	42.6021	**5.59%**	**293,098.32**
2006	74.9031	37.2883	**14.95%**	**256,539.79**	80.5762	47.8388	**12.29%**	**329,126.23**
2007	86.8638	43.2426	**15.97%**	**297,504.65**	91.7756	54.4880	**13.90%**	**374,871.95**
2008	96.2768	47.9286	**10.84%**	**329,743.76**	99.9380	59.3341	**8.89%**	**408,212.57**
2009	88.1920	43.9038	**-8.40%**	**302,053.68**	90.4175	53.6817	**-9.53%**	**369,324.58**
2010	***99.3194***	49.4432	**12.62%**	**340,164.53**	***100.2085***	59.4947	**10.83%**	**409,317.47**
2011	102.3820	50.9679	**3.08%**	**350,653.80**	100.6818	59.7757	**0.47%**	**411,250.73**
2012	109.9890	54.7548	**7.43%**	**376,707.44**	105.6935	62.7512	**4.98%**	**431,721.82**
2013	110.7442	55.1307	**0.69%**	**379,293.96**	105.4620	62.6137	**-0.22%**	**430,776.22**
2014	116.3386	57.9157	**5.05%**	**398,454.53**	109.2708	64.8751	**3.61%**	**446,333.86**
2015	123.4238	61.4429	**6.09%**	**422,721.03**	114.6922	68.0938	**4.96%**	**468,478.43**
2016	139.4587	69.4254	**12.99%**	**477,639.85**	127.6241	75.7716	**11.28%**	**521,300.82**
2017	163.9310	81.6082	**17.55%**	**561,456.39**	147.2050	87.3969	**15.34%**	**601,282.10**
2018	176.1313	87.6818	**7.44%**	**603,241.87**	154.9742	92.0096	**5.28%**	**633,016.63**
2019	177.1623	88.1950	**0.59%**	**606,773.00**	153.4089	91.0802	**-1.01%**	**626,622.91**
2020	184.7230	91.9589	**4.27%**	**632,668.06**	157.1135	93.2797	**2.41%**	**641,754.94**
2021	200.8756	***100.0000***	**8.74%**	**687,990.00**	168.4327	***100.0000***	**7.20%**	**687,990.00**
2022	251.7939	125.3482	**25.35%**	**862,382.91**	199.4938	118.4413	**18.44%**	**814,863.98**

(*) The median house price in dollars was calculated by multiplying the converted House Price Index with reference in 2021 to the MLS® Average Price in that year, according to CMHC [CMHC, (2022)] for both series.
Source: BIS (2022) and author's calculations, using CMHC, (2022).

Table 5 - Contents of a Sample Input CSV File

Tick	Year	Inflation	5Y Fixed Interest	Income	HPI
1	**1975**	**10.69**	**11.38**	**11,600.00**	*39,174.98*
2	**1976**	**7.24**	**11.82**	**12,440.00**	*45,319.35*
3	**1977**	**8.04**	**10.30**	**13,660.73**	*48,056.24*
4	**1978**	**8.93**	**10.49**	**14,907.15**	*49,253.94*
5	**1979**	**9.29**	**12.05**	**16,087.59**	*54,799.29*
6	**1980**	**10.00**	**14.45**	**18,145.99**	*59,061.99*
7	**1981**	**12.50**	**18.35**	**19,872.26**	*70,623.27*
8	**1982**	**10.91**	**18.15**	**21,358.91**	*71,008.23*
9	**1983**	**5.83**	**13.28**	**21,798.10**	*72,847.09*
10	**1984**	**4.30**	**13.60**	**22,957.23**	*72,063.12*
11	**1985**	**3.96**	**12.17**	**23,958.39**	*73,531.40*
12	**1986**	**4.13**	**11.21**	**24,803.50**	*83,410.68*
13	**1987**	**4.42**	**11.16**	**25,800.00**	*103,283.30*
14	**1988**	**3.94**	**11.62**	**27,492.55**	*117,596.19*
15	**1989**	**5.06**	**12.06**	**29,319.42**	*144,339.96*
16	**1990**	**4.81**	**13.32**	**29,471.53**	*140,177.27*
17	**1991**	**5.61**	**11.20**	**29,493.72**	*138,951.47*
18	**1992**	**1.45**	**9.54**	**30,227.74**	*139,707.02*
19	**1993**	**1.90**	**8.78**	**29,991.24**	*145,423.61*
20	**1994**	**0.12**	**9.41**	**30,213.94**	*150,441.51*
21	**1995**	**2.22**	**9.20**	**30,883.80**	*144,582.44*
22	**1996**	**1.48**	**7.98**	**30,822.99**	*140,761.91*
23	**1997**	**1.69**	**7.08**	**31,211.09**	*144,297.14*
24	**1998**	**1.00**	**6.94**	**32,388.18**	*143,812.51*
25	**1999**	**1.75**	**7.52**	**34,447.59**	*144,924.60*
26	**2000**	**2.69**	**8.34**	**35,583.50**	*152,195.09*
27	**2001**	**2.52**	**7.43**	**37,977.81**	*156,029.67*
28	**2002**	**2.25**	**7.00**	**38,759.12**	*172,580.77*
29	**2003**	**2.80**	**6.42**	**39,769.34**	*187,449.53*
30	**2004**	**1.85**	**6.25**	**40,810.07**	*206,980.00*
31	**2005**	**2.20**	**5.98**	**42,643.80**	*223,182.43*
32	**2006**	**1.96**	**6.64**	**44,436.35**	*256,539.79*
33	**2007**	**2.20**	**7.01**	**46,634.67**	*297,504.65*
34	**2008**	**2.33**	**7.10**	**48,388.39**	*329,743.76*
35	**2009**	**0.26**	**5.70**	**48,432.12**	*302,053.68*
36	**2010**	**1.84**	**5.57**	**48,895.99**	*340,164.53*
37	**2011**	**2.92**	**5.39**	**50,147.96**	*350,653.80*
38	**2012**	**1.50**	**5.27**	**52,233.28**	*376,707.44*
39	**2013**	**0.90**	**5.23**	**53,063.94**	*379,293.96*
40	**2014**	**1.95**	**4.89**	**55,380.44**	*398,454.53*
41	**2015**	**1.12**	**4.67**	**55,814.89**	*422,721.03*
42	**2016**	**1.42**	**4.66**	**56,608.47**	*477,639.85*
43	**2017**	**1.56**	**4.77**	**58,917.96**	*561,456.39*
44	**2018**	**2.30**	**5.27**	**60,468.18**	*603,241.87*
45	**2019**	**1.95**	**5.27**	**61,944.53**	*606,773.00*
46	**2020**	**0.74**	**4.95**	**66,800.00**	*632,668.06*
47	**2021**	**3.36**	**4.79**	**69,042.92**	*687,990.00*
48	**2022**	**8.12**	**5.35**	**74,650.22**	*862,382.91*

Table 6 - Contents of Sample Output CSV Files

		Median House Prices		***Median House Prices Variation***	
Tick	**Year**	**(*) ABM: Sold**	**(*) RW: HPI**	**(*) ABM: Sold**	**(*) RW: HPI**
1	**1975**	**48,278.46**	**39,174.98**	**0.00**	**0.00**
2	**1976**	**56,925.24**	**45,319.35**	**0.43**	**15.68**
3	**1977**	**65,785.41**	**48,056.24**	**17.91**	**6.04**
4	**1978**	**72,019.15**	**49,253.94**	**15.56**	**2.49**
5	**1979**	**76,906.89**	**54,799.29**	**9.48**	**11.26**
6	**1980**	**80,033.95**	**59,061.99**	**6.79**	**7.78**
7	**1981**	**84,166.03**	**70,623.27**	**4.07**	**19.57**
8	**1982**	**86,285.54**	**71,008.23**	**5.16**	**0.55**
9	**1983**	**87,457.08**	**72,847.09**	**2.52**	**2.59**
10	**1984**	**97,648.76**	**72,063.12**	**1.36**	**-1.08**
11	**1985**	**109,766.14**	**73,531.40**	**11.65**	**2.04**
12	**1986**	**116,609.06**	**83,410.68**	**12.41**	**13.44**
13	**1987**	**123,412.00**	**103,283.30**	**6.23**	**23.83**
14	**1988**	**138,396.07**	**117,596.19**	**5.83**	**13.86**
15	**1989**	**139,858.05**	**144,339.96**	**12.14**	**22.74**
16	**1990**	**144,131.49**	**140,177.27**	**1.06**	**-2.88**
17	**1991**	**144,863.26**	**138,951.47**	**3.06**	**-0.87**
18	**1992**	**161,429.68**	**139,707.02**	**0.51**	**0.54**
19	**1993**	**174,576.74**	**145,423.61**	**11.44**	**4.09**
20	**1994**	**167,779.57**	**150,441.51**	**8.14**	**3.45**
21	**1995**	**163,125.22**	**144,582.44**	**-3.89**	**-3.89**
22	**1996**	**181,636.76**	**140,761.91**	**-2.77**	**-2.64**
23	**1997**	**198,314.59**	**144,297.14**	**11.35**	**2.51**
24	**1998**	**213,777.92**	**143,812.51**	**9.18**	**-0.34**
25	**1999**	**223,365.00**	**144,924.60**	**7.80**	**0.77**
26	**2000**	**221,457.71**	**152,195.09**	**4.48**	**5.02**
27	**2001**	**235,237.52**	**156,029.67**	**-0.85**	**2.52**
28	**2002**	**259,289.58**	**172,580.77**	**6.22**	**10.61**
29	**2003**	**275,652.55**	**187,449.53**	**10.22**	**8.62**
30	**2004**	**299,093.49**	**206,980.00**	**6.31**	**10.42**
31	**2005**	**304,377.68**	**223,182.43**	**8.50**	**7.83**
32	**2006**	**307,125.35**	**256,539.79**	**1.77**	**14.95**
33	**2007**	**317,142.42**	**297,504.65**	**0.90**	**15.97**
34	**2008**	**315,267.85**	**329,743.76**	**3.26**	**10.84**
35	**2009**	**334,031.61**	**302,053.68**	**-0.59**	**-8.40**
36	**2010**	**389,026.13**	**340,164.53**	**5.95**	**12.62**
37	**2011**	**396,850.01**	**350,653.80**	**16.46**	**3.08**
38	**2012**	**404,821.43**	**376,707.44**	**2.01**	**7.43**
39	**2013**	**423,672.37**	**379,293.96**	**2.01**	**0.69**
40	**2014**	**475,563.90**	**398,454.53**	**4.66**	**5.05**
41	**2015**	**503,297.53**	**422,721.03**	**12.25**	**6.09**
42	**2016**	**563,765.94**	**477,639.85**	**5.83**	**12.99**
43	**2017**	**574,356.34**	**561,456.39**	**12.01**	**17.55**
44	**2018**	**536,037.40**	**603,241.87**	**1.88**	**7.44**
45	**2019**	**525,751.21**	**606,773.00**	**-6.67**	**0.59**
46	**2020**	**547,153.72**	**632,668.06**	**-1.92**	**4.27**
47	**2021**	**633,645.80**	**687,990.00**	**4.07**	**8.74**
48	**2022**	**656,740.30**	**862,382.91**	**15.81**	**25.35**

(*) The columns with header "ABM: Sold" contain data generated by the model. The columns with header "RW: HPI" contain benchmark data from the real world. In this example, the correlation between the benchmark data and the results of ABM simulation is 0.9668, indicating a strong positive correlation between the real-world house prices and ABM house prices. Contents of outputs files were combined into a single table for better presentation.

Table 7 - ABM Simulations Results: Residential Property Prices (1/3)

Year	Benchmark	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6	Simulation 7
1975	***39,174.98***	48,560.70	47,283.67	47,840.94	48,314.15	47,638.58	48,205.49	47,770.67
1976	***45,319.35***	58,833.69	55,756.10	57,110.48	58,581.02	58,901.68	54,278.83	55,858.05
1977	***48,056.24***	65,561.00	64,636.17	61,886.79	65,185.19	65,651.55	63,078.50	67,444.00
1978	***49,253.94***	75,161.73	69,673.66	71,197.56	73,376.22	74,252.69	72,270.85	74,563.09
1979	***54,799.29***	80,330.17	70,682.54	80,167.91	76,476.30	76,545.16	74,793.33	78,614.19
1980	***59,061.99***	81,468.32	71,325.96	80,317.85	78,407.92	79,229.94	77,633.33	81,011.13
1981	***70,623.27***	80,463.52	74,261.97	79,021.75	80,485.99	78,028.79	75,797.66	80,641.96
1982	***71,008.23***	78,613.91	79,652.04	78,986.87	82,170.20	78,926.98	78,186.26	81,078.46
1983	***72,847.09***	80,333.78	80,365.72	84,551.44	86,513.74	81,661.94	84,864.99	89,330.27
1984	***72,063.12***	88,068.03	85,616.31	97,058.93	91,067.16	92,536.44	98,085.15	101,828.83
1985	***73,531.40***	97,344.07	94,927.47	101,096.03	97,507.68	99,360.15	105,138.39	105,549.22
1986	***83,410.68***	105,739.37	96,960.44	112,014.33	107,101.82	118,299.27	118,857.41	113,796.31
1987	***103,283.30***	113,825.22	110,651.19	123,775.02	114,605.82	124,921.11	135,879.11	130,360.36
1988	***117,596.19***	122,986.45	118,498.76	120,535.87	112,930.86	128,227.18	136,030.07	145,787.84
1989	***144,339.96***	134,189.27	117,142.53	131,595.92	116,750.18	143,293.92	139,645.60	148,865.87
1990	***140,177.27***	131,804.25	123,125.00	139,868.91	118,584.80	143,505.58	146,329.04	148,078.03
1991	***138,951.47***	131,777.49	130,056.77	145,223.16	132,561.21	146,984.48	151,213.08	153,675.71
1992	***139,707.02***	143,502.14	141,475.07	154,824.52	147,479.51	160,250.91	155,594.31	162,921.92
1993	***145,423.61***	151,950.89	152,632.69	164,937.53	153,038.19	174,283.98	168,712.25	181,225.62
1994	***150,441.51***	159,379.31	150,558.65	162,620.12	150,102.34	181,238.48	177,755.63	180,173.33
1995	***144,582.44***	162,790.18	154,012.16	162,513.00	151,727.79	176,534.07	188,210.77	181,861.90
1996	***140,761.91***	168,041.54	165,356.89	176,389.99	172,317.08	179,382.90	198,759.13	194,704.45
1997	***144,297.14***	180,207.44	184,513.64	194,946.76	179,706.70	191,979.41	206,497.27	202,835.39
1998	***143,812.51***	195,948.14	189,428.86	202,771.24	191,366.43	203,707.40	227,808.08	213,676.42
1999	***144,924.60***	195,653.14	188,661.30	209,599.09	191,366.43	211,240.05	221,750.65	215,565.67
2000	***152,195.09***	187,966.71	188,913.94	223,319.85	192,881.40	216,895.50	227,374.81	210,725.24
2001	***156,029.67***	192,076.60	199,309.55	232,100.02	200,196.61	230,449.22	255,711.05	218,836.05
2002	***172,580.77***	215,397.51	220,520.54	248,753.85	236,284.87	242,523.15	259,754.48	248,937.72
2003	***187,449.53***	237,690.53	234,261.01	258,305.37	242,703.45	249,233.58	276,662.50	267,880.76
2004	***206,980.00***	252,127.63	258,984.85	282,587.18	272,138.53	271,523.77	292,049.96	292,578.74
2005	***223,182.43***	277,560.91	264,180.98	292,411.73	301,891.84	300,920.79	313,360.59	293,914.87
2006	***256,539.79***	292,082.80	290,247.11	287,866.43	294,420.55	327,189.33	311,300.86	289,701.09
2007	***297,504.65***	298,320.99	288,900.66	287,732.19	285,869.51	342,748.89	319,269.82	291,711.19
2008	***329,743.76***	303,299.31	269,853.36	302,162.86	307,944.94	344,084.36	328,806.78	312,594.87
2009	***302,053.68***	329,721.68	295,958.28	329,764.25	330,601.85	358,295.95	352,697.16	335,517.62
2010	***340,164.53***	365,623.07	339,743.14	350,550.32	358,074.76	401,779.07	385,269.89	373,289.50
2011	***350,653.80***	388,437.41	393,384.86	356,317.38	387,870.22	407,549.65	389,584.46	381,223.25
2012	***376,707.44***	402,258.65	411,270.68	408,721.56	388,245.72	466,593.99	447,276.47	388,632.51
2013	***379,293.96***	455,129.24	427,957.40	409,235.60	414,418.78	482,267.85	453,108.97	390,388.17
2014	***398,454.53***	463,017.62	454,736.27	421,307.69	446,499.79	469,838.30	458,737.36	438,865.85
2015	***422,721.03***	454,906.30	490,398.58	482,439.64	491,150.49	532,124.34	541,998.18	483,105.22
2016	***477,639.85***	489,141.92	527,373.40	492,070.63	517,211.62	560,509.64	562,739.59	501,329.30
2017	***561,456.39***	529,879.52	564,260.04	506,270.92	521,303.39	551,168.16	544,251.64	529,692.62
2018	***603,241.87***	533,089.84	542,296.57	497,896.43	504,532.13	546,184.99	543,082.81	553,911.92
2019	***606,773.00***	552,319.52	525,066.63	513,499.69	528,279.48	516,843.55	551,825.98	539,578.88
2020	***632,668.06***	617,311.37	533,718.16	571,380.27	583,479.32	583,849.48	596,790.08	551,174.20
2021	***687,990.00***	651,430.84	599,022.03	628,104.82	584,083.45	646,174.88	654,839.64	645,054.09
2022	***862,382.91***	642,223.97	625,676.70	635,796.54	619,763.60	686,079.02	675,090.33	654,526.97

Table 8 - ABM Simulations Results: Residential Property Prices (2/3)

Year	Simulation 8	Simulation 9	Simulation 10	Simulation 11	Simulation 12	Simulation 13	Simulation 14	Simulation 15
1975	47,844.57	47,430.22	47,323.21	47,543.90	48,247.47	48,092.54	47,737.40	47,963.90
1976	58,851.98	59,469.35	55,400.61	59,516.18	56,995.89	57,411.47	56,483.48	54,356.09
1977	62,727.00	62,353.91	60,658.25	63,156.23	64,208.31	66,243.56	65,744.79	61,284.67
1978	69,963.47	67,542.62	66,258.67	72,280.95	74,068.11	69,999.87	75,339.43	69,837.21
1979	72,903.30	72,287.78	68,548.19	72,709.62	78,643.21	73,667.51	79,804.88	73,100.60
1980	74,442.29	74,005.42	74,450.74	77,052.37	82,146.17	79,382.97	80,289.46	76,748.16
1981	75,864.63	75,408.62	73,593.84	77,699.01	84,338.49	80,643.59	79,842.08	81,137.62
1982	80,064.96	77,637.72	78,705.77	79,041.32	86,332.81	82,643.59	81,591.42	81,491.32
1983	85,928.98	83,879.65	84,456.83	83,305.64	92,091.02	89,067.68	84,956.39	87,978.24
1984	94,460.58	93,101.47	90,658.14	91,392.96	92,721.04	101,238.55	92,155.98	100,112.53
1985	104,362.16	98,857.02	97,994.53	94,421.51	99,904.58	102,718.52	102,496.86	106,915.43
1986	111,243.99	104,590.67	106,674.76	112,190.69	113,728.58	109,282.37	112,002.66	111,390.45
1987	115,549.26	115,979.57	119,633.83	123,398.58	122,228.94	118,939.88	123,349.59	118,269.58
1988	122,850.82	127,874.13	124,351.93	134,283.66	130,234.88	128,112.87	124,783.29	135,868.97
1989	127,824.17	137,291.85	122,271.74	146,556.08	134,098.34	132,300.88	131,659.74	138,511.28
1990	132,598.38	133,373.86	126,270.84	143,035.70	126,662.23	133,719.55	137,719.31	141,511.95
1991	133,093.73	138,998.72	130,749.58	142,163.69	135,534.65	145,529.39	145,993.08	148,701.60
1992	152,828.52	159,462.97	141,604.70	154,175.73	150,102.32	156,374.64	157,068.27	158,500.66
1993	162,279.16	163,367.90	152,932.50	170,477.89	153,413.21	165,909.63	161,885.94	163,121.74
1994	159,118.24	153,276.68	146,195.18	171,990.37	164,114.00	171,197.51	166,499.29	168,879.53
1995	158,330.92	162,629.14	151,559.11	174,953.59	168,739.75	162,577.30	168,634.30	168,746.09
1996	172,678.00	173,909.87	158,102.02	188,867.95	175,128.04	162,794.71	181,983.35	183,862.44
1997	181,769.42	184,112.33	178,194.32	195,528.70	187,987.51	185,795.93	182,759.74	193,436.51
1998	196,072.28	201,135.14	199,480.09	207,745.88	201,250.25	190,440.07	202,184.35	199,451.81
1999	205,043.46	216,698.70	200,428.79	218,548.36	193,968.19	200,039.23	216,347.86	214,193.27
2000	213,632.69	210,997.53	200,155.56	223,373.83	200,435.31	205,695.21	210,196.28	214,234.96
2001	216,226.52	227,632.80	230,486.27	230,649.83	219,218.98	218,699.67	218,403.17	216,545.62
2002	228,267.16	249,381.93	236,404.85	230,875.19	229,603.17	241,075.98	249,931.11	233,640.83
2003	252,434.43	267,400.07	247,067.38	261,118.26	257,020.86	261,724.62	267,940.58	252,948.76
2004	265,751.78	282,575.25	270,028.99	274,383.22	276,906.16	276,424.97	271,282.16	286,906.32
2005	277,649.27	300,442.14	285,630.56	292,581.02	285,751.77	294,799.78	279,715.09	281,363.57
2006	286,856.08	340,450.90	274,312.55	308,517.66	299,369.70	311,570.76	300,171.11	286,105.44
2007	275,332.09	332,109.66	309,545.51	301,458.14	298,650.47	309,073.70	281,066.25	302,178.21
2008	294,720.57	344,592.62	347,417.87	318,259.98	305,310.96	305,765.05	293,862.17	302,161.95
2009	334,515.16	363,607.17	351,791.56	342,095.91	319,505.07	333,112.38	343,700.63	322,733.89
2010	351,138.64	404,013.06	376,494.81	377,880.66	367,370.40	360,493.81	357,468.27	385,893.50
2011	378,576.51	401,750.18	393,168.47	385,167.26	406,586.03	397,912.32	372,736.20	390,990.70
2012	403,411.09	413,965.98	439,361.02	445,488.55	440,646.85	396,221.72	398,690.33	393,155.43
2013	416,513.70	445,826.29	461,843.11	468,652.40	466,365.45	432,108.60	434,992.25	415,395.53
2014	457,086.87	467,217.80	460,886.90	489,168.63	499,502.77	465,509.20	458,153.24	437,347.74
2015	494,362.72	496,507.50	523,890.30	504,877.40	518,155.76	494,760.40	495,152.27	478,267.71
2016	559,574.24	520,007.97	561,317.86	510,845.58	562,725.33	492,381.56	478,325.47	502,623.92
2017	595,674.85	545,649.55	549,033.81	569,715.27	570,632.59	518,608.26	514,679.95	532,606.37
2018	559,350.02	557,936.01	530,975.88	581,714.20	539,530.41	569,445.82	535,177.15	569,985.86
2019	560,840.71	552,402.01	536,219.06	565,306.35	575,330.89	562,618.76	531,824.19	557,272.45
2020	584,385.84	598,511.36	611,442.00	602,938.36	611,324.38	594,299.29	580,895.57	578,358.04
2021	642,960.12	648,048.75	650,744.12	647,521.32	651,787.56	626,762.48	620,174.93	617,338.91
2022	655,610.98	648,030.45	673,121.01	645,736.58	678,957.22	660,995.64	639,860.99	641,752.73

Table 9 - ABM Simulations Results: Residential Property Prices (3/3)

Year	Simulation 16	Simulation 17	Simulation 18	Simulation 19	Simulation 20	*(*)* Mean	Std. Dev.	CV
1975	47,780.88	48,287.85	47,878.06	47,865.80	48,358.35	***47,898.42***	***356.72***	***0.74%***
1976	59,116.23	56,796.11	58,655.03	58,194.34	56,519.94	***57,354.33***	***1,654.26***	***2.88%***
1977	65,525.47	64,535.73	64,869.12	62,382.26	63,411.45	***64,027.20***	***1,808.92***	***2.83%***
1978	72,498.26	69,224.32	70,986.64	73,310.64	71,070.64	***71,643.83***	***2,511.57***	***3.51%***
1979	76,320.85	72,979.28	72,317.57	76,108.50	72,428.97	***74,971.49***	***3,345.41***	***4.46%***
1980	77,676.51	78,538.44	76,910.66	77,266.93	73,213.74	***77,575.91***	***2,922.47***	***3.77%***
1981	79,578.21	79,815.64	80,137.21	79,641.03	73,372.12	***78,488.69***	***2,901.62***	***3.70%***
1982	81,666.76	79,718.36	83,537.45	81,883.41	73,891.21	***80,291.04***	***2,595.06***	***3.23%***
1983	86,183.99	83,980.87	90,593.25	86,573.39	78,450.19	***85,253.40***	***3,512.80***	***4.12%***
1984	91,297.73	93,166.99	99,825.21	93,986.65	83,820.18	***93,609.94***	***4,919.61***	***5.26%***
1985	96,673.85	98,085.46	105,622.09	100,916.65	93,877.77	***100,188.47***	***3,964.88***	***3.96%***
1986	111,582.68	109,734.16	116,154.69	115,100.49	107,058.92	***110,675.20***	***5,106.30***	***4.61%***
1987	123,590.88	120,013.72	126,844.45	129,028.15	107,989.86	***120,941.71***	***6,837.58***	***5.65%***
1988	126,754.39	117,606.92	123,712.03	131,283.32	124,652.51	***126,868.34***	***7,469.47***	***5.89%***
1989	135,120.04	126,255.30	126,948.38	134,376.14	129,721.11	***132,720.92***	***8,572.87***	***6.46%***
1990	132,007.06	130,415.83	137,899.22	137,851.81	135,094.89	***134,972.81***	***7,721.35***	***5.72%***
1991	137,751.52	139,771.99	150,383.78	141,390.97	138,242.01	***140,989.83***	***7,291.29***	***5.17%***
1992	157,055.96	160,090.67	161,015.99	155,935.63	156,177.71	***154,322.11***	***6,361.72***	***4.12%***
1993	161,781.89	165,209.80	167,307.25	166,709.93	168,137.10	***163,465.76***	***7,762.86***	***4.75%***
1994	168,052.25	161,128.13	167,632.82	170,447.33	159,506.12	***164,493.27***	***9,844.43***	***5.98%***
1995	165,052.62	163,846.78	165,543.63	172,937.43	170,521.24	***166,586.09***	***9,419.40***	***5.65%***
1996	177,413.43	169,972.70	173,271.62	178,564.11	181,383.03	***176,644.16***	***10,043.89***	***5.69%***
1997	191,538.89	190,723.49	195,594.42	199,682.97	195,131.13	***190,147.10***	***7,978.23***	***4.20%***
1998	196,163.34	194,952.02	213,046.70	218,759.39	212,445.60	***202,891.68***	***10,024.88***	***4.94%***
1999	202,526.77	198,800.94	219,967.56	221,062.30	211,283.66	***207,637.27***	***10,564.00***	***5.09%***
2000	198,102.64	207,813.18	218,469.90	219,406.91	206,083.57	***208,833.75***	***11,385.95***	***5.45%***
2001	223,103.85	214,061.50	214,917.50	228,587.49	219,206.01	***220,320.92***	***13,741.84***	***6.24%***
2002	257,988.76	234,291.51	228,073.49	229,710.21	232,171.82	***237,679.41***	***11,875.94***	***5.00%***
2003	252,019.53	248,875.22	261,423.80	252,659.47	253,200.37	***255,128.53***	***10,607.86***	***4.16%***
2004	259,629.41	260,771.47	269,332.55	292,996.86	296,287.44	***275,263.36***	***12,634.36***	***4.59%***
2005	302,575.31	273,292.64	313,647.35	305,389.49	297,344.53	***291,721.21***	***13,348.91***	***4.58%***
2006	313,796.88	273,294.23	326,994.90	319,127.01	308,620.01	***302,099.77***	***18,009.11***	***5.96%***
2007	280,388.30	264,357.50	324,444.78	329,343.13	299,649.68	***301,107.53***	***20,536.24***	***6.82%***
2008	286,331.70	280,526.62	332,769.34	318,714.72	288,442.08	***309,381.10***	***21,746.18***	***7.03%***
2009	340,147.68	319,317.16	369,140.48	338,140.53	304,185.64	***335,727.50***	***18,442.78***	***5.49%***
2010	361,097.86	358,995.55	381,228.71	380,798.78	358,323.06	***369,776.34***	***16,876.15***	***4.56%***
2011	361,765.75	363,237.51	415,324.01	398,370.16	378,927.64	***387,444.00***	***15,717.12***	***4.06%***
2012	395,539.98	388,474.40	441,857.72	416,569.05	393,923.60	***414,015.27***	***24,032.10***	***5.80%***
2013	419,426.09	406,393.32	448,420.06	438,588.90	413,164.46	***435,009.81***	***24,616.75***	***5.66%***
2014	431,906.37	435,622.15	488,749.00	444,177.36	431,721.77	***456,002.63***	***20,801.33***	***4.56%***
2015	483,006.48	478,363.60	548,804.30	456,577.14	452,273.03	***495,056.07***	***27,046.05***	***5.46%***
2016	527,461.48	494,935.45	552,924.42	522,247.70	484,666.35	***521,020.67***	***29,554.00***	***5.67%***
2017	519,765.25	557,722.54	561,101.10	537,795.67	490,746.94	***540,527.92***	***25,349.27***	***4.69%***
2018	518,684.15	528,569.45	570,175.05	535,273.77	516,582.75	***541,719.76***	***22,524.18***	***4.16%***
2019	524,482.58	541,798.78	566,733.45	528,478.06	502,099.02	***541,641.00***	***20,173.80***	***3.72%***
2020	584,129.39	604,539.81	596,587.84	626,619.66	524,824.95	***586,827.97***	***26,151.11***	***4.46%***
2021	671,042.20	619,277.48	634,131.93	669,143.60	585,710.27	***634,667.67***	***24,584.67***	***3.87%***
2022	631,028.47	641,663.15	684,467.08	664,040.06	609,491.85	***650,695.67***	***21,623.45***	***3.32%***

(*) The correlation between the benchmark data and the mean of the results of ABM simulations is 0.9749, indicating a strong positive correlation between the real-world house prices and ABM house prices

Table 10 - ABM Simulations Results: Residential Property Prices Variation (1/3)

Year	Benchmark	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6	Simulation 7
1975	***0.00%***	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1976	***15.68%***	0.42%	0.24%	0.56%	0.44%	0.52%	0.21%	0.25%
1977	***6.04%***	21.15%	17.92%	19.38%	21.25%	23.64%	12.60%	16.93%
1978	***2.49%***	11.43%	15.93%	8.36%	11.27%	11.46%	16.21%	20.74%
1979	***11.26%***	14.64%	7.79%	15.04%	12.57%	13.10%	14.57%	10.56%
1980	***7.78%***	6.88%	1.45%	12.60%	4.22%	3.09%	3.49%	5.43%
1981	***19.57%***	1.42%	0.91%	0.19%	2.53%	3.51%	3.80%	3.05%
1982	***0.55%***	-1.23%	4.12%	-1.61%	2.65%	-1.52%	-2.36%	-0.46%
1983	***2.59%***	-2.30%	7.26%	-0.04%	2.09%	1.15%	3.15%	0.54%
1984	***-1.08%***	2.19%	0.90%	7.04%	5.29%	3.47%	8.54%	10.18%
1985	***2.04%***	9.63%	6.53%	14.79%	5.26%	13.32%	15.58%	13.99%
1986	***13.44%***	10.53%	10.88%	4.16%	7.07%	7.37%	7.19%	3.65%
1987	***23.83%***	8.62%	2.14%	10.80%	9.84%	19.06%	13.05%	7.81%
1988	***13.86%***	7.65%	14.12%	10.50%	7.01%	5.60%	14.32%	14.56%
1989	***22.74%***	8.05%	7.09%	-2.62%	-1.46%	2.65%	0.11%	11.83%
1990	***-2.88%***	9.11%	-1.14%	9.18%	3.38%	11.75%	2.66%	2.11%
1991	***-0.87%***	-1.78%	5.11%	6.29%	1.57%	0.15%	4.79%	-0.53%
1992	***0.54%***	-0.02%	5.63%	3.83%	11.79%	2.42%	3.34%	3.78%
1993	***4.09%***	8.90%	8.78%	6.61%	11.25%	9.03%	2.90%	6.02%
1994	***3.45%***	5.89%	7.89%	6.53%	3.77%	8.76%	8.43%	11.23%
1995	***-3.89%***	4.89%	-1.36%	-1.41%	-1.92%	3.99%	5.36%	-0.58%
1996	***-2.64%***	2.14%	2.29%	-0.07%	1.08%	-2.60%	5.88%	0.94%
1997	***2.51%***	3.23%	7.37%	8.54%	13.57%	1.61%	5.60%	7.06%
1998	***-0.34%***	7.24%	11.59%	10.52%	4.29%	7.02%	3.89%	4.18%
1999	***0.77%***	8.73%	2.66%	4.01%	6.49%	6.11%	10.32%	5.34%
2000	***5.02%***	-0.15%	-0.41%	3.37%	0.00%	3.70%	-2.66%	0.88%
2001	***2.52%***	-3.93%	0.13%	6.55%	0.79%	2.68%	2.54%	-2.25%
2002	***10.61%***	2.19%	5.50%	3.93%	3.79%	6.25%	12.46%	3.85%
2003	***8.62%***	12.14%	10.64%	7.18%	18.03%	5.24%	1.58%	13.76%
2004	***10.42%***	10.35%	6.23%	3.84%	2.72%	2.77%	6.51%	7.61%
2005	***7.83%***	6.07%	10.55%	9.40%	12.13%	8.94%	5.56%	9.22%
2006	***14.95%***	10.09%	2.01%	3.48%	10.93%	10.83%	7.30%	0.46%
2007	***15.97%***	5.23%	9.87%	-1.55%	-2.47%	8.73%	-0.66%	-1.43%
2008	***10.84%***	2.14%	-0.46%	-0.05%	-2.90%	4.76%	2.56%	0.69%
2009	***-8.40%***	1.67%	-6.59%	5.02%	7.72%	0.39%	2.99%	7.16%
2010	***12.62%***	8.71%	9.67%	9.13%	7.36%	4.13%	7.27%	7.33%
2011	***3.08%***	10.89%	14.79%	6.30%	8.31%	12.14%	9.24%	11.26%
2012	***7.43%***	6.24%	15.79%	1.65%	8.32%	1.44%	1.12%	2.13%
2013	***0.69%***	3.56%	4.55%	14.71%	0.10%	14.49%	14.81%	1.94%
2014	***5.05%***	13.14%	4.06%	0.13%	6.74%	3.36%	1.30%	0.45%
2015	***6.09%***	1.73%	6.26%	2.95%	7.74%	-2.58%	1.24%	12.42%
2016	***12.99%***	-1.75%	7.84%	14.51%	10.00%	13.26%	18.15%	10.08%
2017	***17.55%***	7.53%	7.54%	2.00%	5.31%	5.33%	3.83%	3.77%
2018	***7.44%***	8.33%	6.99%	2.89%	0.79%	-1.67%	-3.29%	5.66%
2019	***0.59%***	0.61%	-3.89%	-1.65%	-3.22%	-0.90%	-0.21%	4.57%
2020	***4.27%***	3.61%	-3.18%	3.13%	4.71%	-5.37%	1.61%	-2.59%
2021	***8.74%***	11.77%	1.65%	11.27%	10.45%	12.96%	8.15%	2.15%
2022	***25.35%***	5.53%	12.24%	9.93%	0.10%	10.67%	9.73%	17.03%

Table 11 - ABM Simulations Results: Residential Property Prices Variation (2/3)

Year	Simulation 8	Simulation 9	Simulation 10	Simulation 11	Simulation 12	Simulation 13	Simulation 14	Simulation 15
1975	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1976	0.66%	0.62%	0.33%	0.45%	0.45%	0.34%	0.25%	0.25%
1977	23.01%	25.38%	17.07%	25.18%	18.13%	19.38%	18.32%	13.33%
1978	6.58%	4.85%	9.49%	6.12%	12.65%	15.38%	16.40%	12.75%
1979	11.54%	8.32%	9.23%	14.45%	15.36%	5.67%	14.59%	13.96%
1980	4.20%	7.03%	3.46%	0.59%	6.18%	5.24%	5.93%	4.67%
1981	2.11%	2.38%	8.61%	5.97%	4.45%	7.76%	0.61%	4.99%
1982	1.91%	1.90%	-1.15%	0.84%	2.67%	1.59%	-0.56%	5.72%
1983	5.54%	2.96%	6.95%	1.73%	2.36%	2.48%	2.19%	0.44%
1984	7.32%	8.04%	7.31%	5.40%	6.67%	7.77%	4.12%	7.96%
1985	9.93%	10.99%	7.34%	9.71%	0.68%	13.66%	8.47%	13.79%
1986	10.48%	6.18%	8.09%	3.31%	7.75%	1.46%	11.22%	6.80%
1987	6.59%	5.80%	8.86%	18.82%	13.84%	6.39%	9.27%	4.19%
1988	3.87%	10.89%	12.15%	9.99%	7.47%	8.84%	10.13%	6.18%
1989	6.32%	10.26%	3.94%	8.82%	6.55%	7.71%	1.16%	14.88%
1990	4.05%	7.36%	-1.67%	9.14%	2.97%	3.27%	5.51%	1.94%
1991	3.73%	-2.85%	3.27%	-2.40%	-5.55%	1.07%	4.60%	2.17%
1992	0.37%	4.22%	3.55%	-0.61%	7.00%	8.83%	6.01%	5.08%
1993	14.83%	14.72%	8.30%	8.45%	10.75%	7.45%	7.59%	6.59%
1994	6.18%	2.45%	8.00%	10.57%	2.21%	6.10%	3.07%	2.92%
1995	-1.95%	-6.18%	-4.41%	0.89%	6.98%	3.19%	2.85%	3.53%
1996	-0.49%	6.10%	3.67%	1.72%	2.82%	-5.04%	1.28%	-0.08%
1997	9.06%	6.94%	4.32%	7.95%	3.79%	0.13%	7.92%	8.96%
1998	5.26%	5.87%	12.71%	3.53%	7.34%	14.13%	0.43%	5.21%
1999	7.87%	9.25%	11.95%	6.25%	7.06%	2.50%	10.63%	3.11%
2000	4.58%	7.74%	0.48%	5.20%	-3.62%	5.04%	7.01%	7.39%
2001	4.19%	-2.63%	-0.14%	2.21%	3.33%	2.83%	-2.84%	0.02%
2002	1.21%	7.88%	15.15%	3.26%	9.37%	6.32%	3.90%	1.08%
2003	5.57%	9.55%	2.57%	0.10%	4.74%	10.23%	14.44%	7.89%
2004	10.59%	7.23%	4.51%	13.10%	11.94%	8.57%	7.21%	8.26%
2005	5.28%	5.68%	9.29%	5.08%	7.74%	5.62%	1.25%	13.42%
2006	4.48%	6.32%	5.78%	6.63%	3.19%	6.65%	3.11%	-1.93%
2007	3.32%	13.32%	-3.96%	5.45%	4.77%	5.69%	7.31%	1.69%
2008	-4.02%	-2.45%	12.84%	-2.29%	-0.24%	-0.80%	-6.36%	5.62%
2009	7.04%	3.76%	12.23%	5.57%	2.23%	-1.07%	4.55%	-0.01%
2010	13.50%	5.52%	1.26%	7.49%	4.65%	8.94%	16.96%	6.81%
2011	4.97%	11.11%	7.02%	10.46%	14.98%	8.22%	4.01%	19.57%
2012	7.81%	-0.56%	4.43%	1.93%	10.67%	10.38%	4.27%	1.32%
2013	6.56%	3.04%	11.75%	15.66%	8.38%	-0.42%	6.96%	0.55%
2014	3.25%	7.70%	5.12%	5.20%	5.84%	9.06%	9.11%	5.66%
2015	9.74%	4.80%	-0.21%	4.38%	7.11%	7.73%	5.32%	5.28%
2016	8.16%	6.27%	13.67%	3.21%	3.73%	6.28%	8.08%	9.36%
2017	13.19%	4.73%	7.14%	1.18%	8.60%	-0.48%	-3.40%	5.09%
2018	6.45%	4.93%	-2.19%	11.52%	1.41%	5.33%	7.60%	5.97%
2019	-6.10%	2.25%	-3.29%	2.11%	-5.45%	9.80%	3.98%	7.02%
2020	0.27%	-0.99%	0.99%	-2.82%	6.64%	-1.20%	-0.63%	-2.23%
2021	4.20%	8.35%	14.03%	6.66%	6.26%	5.63%	9.23%	3.78%
2022	10.02%	8.28%	6.43%	7.39%	6.62%	5.46%	6.76%	6.74%

Table 12 - ABM Simulations Results: Residential Property Prices Variation (3/3)

Year	Simulation 16	Simulation 17	Simulation 18	Simulation 19	Simulation 20	Mean	Std. Dev.
1975	0.00%	0.00%	0.00%	0.00%	0.00%	***0.00%***	***0.00%***
1976	0.48%	0.41%	0.24%	0.59%	0.42%	***0.41%***	***0.14%***
1977	23.72%	17.62%	22.51%	21.58%	16.88%	***19.75%***	***3.60%***
1978	10.84%	13.63%	10.59%	7.20%	12.19%	***11.70%***	***4.00%***
1979	10.64%	7.27%	9.43%	17.52%	12.08%	***11.92%***	***3.20%***
1980	5.27%	5.42%	1.87%	3.82%	1.91%	***4.64%***	***2.59%***
1981	1.78%	7.62%	6.35%	1.52%	1.08%	***3.53%***	***2.58%***
1982	2.45%	1.63%	4.20%	3.07%	0.22%	***1.20%***	***2.24%***
1983	2.62%	-0.12%	4.24%	2.82%	0.71%	***2.34%***	***2.35%***
1984	5.53%	5.35%	8.45%	5.73%	6.17%	***6.17%***	***2.27%***
1985	5.93%	10.94%	10.19%	8.56%	6.85%	***9.81%***	***3.77%***
1986	5.89%	5.28%	5.81%	7.37%	12.00%	***7.13%***	***2.85%***
1987	15.42%	11.88%	9.97%	14.06%	14.04%	***10.52%***	***4.54%***
1988	10.76%	9.37%	9.20%	12.10%	0.87%	***9.28%***	***3.52%***
1989	2.56%	-2.01%	-2.47%	1.75%	15.43%	***5.03%***	***5.56%***
1990	6.60%	7.35%	2.62%	2.36%	4.07%	***4.63%***	***3.54%***
1991	-2.30%	3.30%	8.63%	2.59%	4.14%	***1.80%***	***3.56%***
1992	4.35%	7.17%	9.05%	2.57%	2.33%	***4.53%***	***3.16%***
1993	14.01%	14.54%	7.07%	10.29%	12.97%	***9.55%***	***3.31%***
1994	3.01%	3.20%	3.91%	6.91%	7.66%	***5.93%***	***2.76%***
1995	3.88%	-2.47%	0.19%	2.24%	-5.13%	***0.63%***	***3.72%***
1996	-1.78%	1.69%	-1.25%	1.46%	6.91%	***1.33%***	***2.94%***
1997	7.49%	3.74%	4.67%	3.25%	6.37%	***6.08%***	***3.07%***
1998	7.96%	12.21%	12.88%	11.83%	7.58%	***7.78%***	***3.83%***
1999	2.41%	2.22%	8.92%	9.55%	8.87%	***6.71%***	***3.09%***
2000	3.24%	1.97%	3.25%	1.05%	-0.55%	***2.38%***	***3.22%***
2001	-2.18%	4.53%	-0.68%	-0.75%	-2.46%	***0.60%***	***2.90%***
2002	12.62%	3.01%	-1.63%	4.18%	6.37%	***5.54%***	***4.22%***
2003	15.64%	9.45%	6.12%	0.49%	5.91%	***8.06%***	***5.07%***
2004	-2.31%	6.22%	14.62%	9.99%	9.06%	***7.45%***	***3.96%***
2005	3.02%	4.78%	3.03%	15.97%	17.02%	***7.95%***	***4.27%***
2006	16.54%	4.80%	16.45%	4.23%	0.36%	***6.08%***	***4.91%***
2007	3.71%	0.00%	4.26%	4.50%	3.79%	***3.58%***	***4.40%***
2008	-10.65%	-3.27%	-0.78%	3.20%	-2.91%	***-0.27%***	***4.87%***
2009	2.12%	6.12%	2.57%	-3.23%	-3.74%	***2.82%***	***4.45%***
2010	18.79%	13.83%	10.93%	6.10%	5.46%	***8.69%***	***4.33%***
2011	6.16%	12.43%	3.27%	12.62%	17.80%	***10.28%***	***4.41%***
2012	0.18%	1.18%	8.94%	4.61%	5.75%	***4.88%***	***4.32%***
2013	9.34%	6.95%	6.39%	4.57%	3.96%	***6.89%***	***5.13%***
2014	6.04%	4.61%	1.49%	5.29%	4.88%	***5.12%***	***3.16%***
2015	2.98%	7.19%	8.99%	1.27%	4.49%	***4.94%***	***3.64%***
2016	11.83%	9.81%	12.29%	2.79%	4.76%	***8.62%***	***4.72%***
2017	9.20%	3.46%	0.75%	14.38%	7.16%	***5.32%***	***4.31%***
2018	-1.46%	12.69%	1.48%	2.98%	1.25%	***3.88%***	***4.44%***
2019	-0.21%	-5.23%	1.62%	-0.47%	5.26%	***0.33%***	***4.32%***
2020	1.12%	2.50%	-0.60%	-1.27%	-2.80%	***0.04%***	***3.00%***
2021	11.37%	11.58%	5.27%	18.57%	4.53%	***8.39%***	***4.37%***
2022	14.88%	2.44%	6.29%	6.79%	11.60%	***8.25%***	***3.94%***

Table 13 - Correlation of Interest Rates and ABM House Prices

	(*) Mortgage Interest Rate		(*) ABM Median House Price			
Year	**ABM Input (%)**	**Normalized (%)**	**ABM Output ($)**	**CPI Ref. 2010**	**Deflated ($)**	**Normalized (%)**
1975	0.1138	**0.6200**	48,278.46	29.00	166,477.44	**0.3720**
1976	0.1182	**0.6440**	56,925.24	31.10	183,039.37	**0.4090**
1977	0.1030	**0.5612**	65,785.41	33.60	195,789.92	**0.4375**
1978	0.1049	**0.5717**	72,019.15	36.60	196,773.63	**0.4397**
1979	0.1205	**0.6568**	76,906.89	40.00	192,267.22	**0.4297**
1980	0.1445	**0.7873**	80,033.95	44.00	181,895.35	**0.4065**
1981	0.1835	**1.0000**	84,166.03	49.50	170,032.39	**0.3800**
1982	0.1815	**0.9890**	86,285.54	54.90	157,168.57	**0.3512**
1983	0.1328	**0.7239**	87,457.08	58.10	150,528.54	**0.3364**
1984	0.1360	**0.7409**	97,648.76	60.60	161,136.56	**0.3601**
1985	0.1217	**0.6631**	109,766.14	63.00	174,231.97	**0.3894**
1986	0.1121	**0.6110**	116,609.06	65.60	177,757.71	**0.3972**
1987	0.1116	**0.6081**	123,412.00	68.50	180,163.51	**0.4026**
1988	0.1162	**0.6330**	138,396.07	71.20	194,376.50	**0.4344**
1989	0.1206	**0.6571**	139,858.05	74.80	186,976.01	**0.4178**
1990	0.1332	**0.7257**	144,131.49	78.40	183,841.19	**0.4108**
1991	0.1120	**0.6105**	144,863.26	82.80	174,955.63	**0.3910**
1992	0.0954	**0.5199**	161,429.68	84.00	192,178.20	**0.4295**
1993	0.0878	**0.4785**	174,576.74	85.60	203,944.79	**0.4558**
1994	0.0941	**0.5128**	167,779.57	85.70	195,775.46	**0.4375**
1995	0.0920	**0.5016**	163,125.22	87.60	186,216.00	**0.4161**
1996	0.0798	**0.4346**	181,636.76	88.90	204,315.82	**0.4566**
1997	0.0708	**0.3858**	198,314.59	90.40	219,374.54	**0.4902**
1998	0.0694	**0.3784**	213,777.92	91.30	234,148.87	**0.5232**
1999	0.0752	**0.4099**	223,365.00	92.90	240,435.95	**0.5373**
2000	0.0834	**0.4547**	221,457.71	95.40	232,135.96	**0.5188**
2001	0.0743	**0.4047**	235,237.52	97.80	240,529.17	**0.5375**
2002	0.0700	**0.3817**	259,289.58	100.00	259,289.58	**0.5794**
2003	0.0642	**0.3499**	275,652.55	102.80	268,144.51	**0.5992**
2004	0.0625	**0.3408**	299,093.49	104.70	285,667.13	**0.6384**
2005	0.0598	**0.3258**	304,377.68	107.00	284,465.13	**0.6357**
2006	0.0664	**0.3616**	307,125.35	109.10	281,508.12	**0.6291**
2007	0.0701	**0.3821**	317,142.42	111.50	284,432.66	**0.6356**
2008	0.0710	**0.3868**	315,267.85	114.10	276,308.37	**0.6175**
2009	0.0570	**0.3103**	334,031.61	114.40	291,985.68	**0.6525**
2010	0.0557	**0.3036**	389,026.13	116.50	333,928.01	**0.7462**
2011	0.0539	**0.2938**	396,850.01	119.90	330,984.16	**0.7396**
2012	0.0527	**0.2873**	404,821.43	121.70	332,638.81	**0.7433**
2013	0.0523	**0.2849**	423,672.37	122.80	345,010.07	**0.7710**
2014	0.0489	**0.2665**	475,563.90	125.20	379,843.37	**0.8488**
2015	0.0467	**0.2546**	503,297.53	126.60	397,549.39	**0.8884**
2016	0.0466	**0.2538**	563,765.94	128.40	439,070.05	**0.9812**
2017	0.0476	**0.2597**	574,356.34	130.40	440,457.32	**0.9843**
2018	0.0527	**0.2869**	536,037.40	133.40	401,827.14	**0.8980**
2019	0.0527	**0.2872**	525,751.21	136.00	386,581.77	**0.8639**
2020	0.0495	**0.2697**	547,153.72	137.00	399,382.28	**0.8925**
2021	0.0479	**0.2610**	633,645.80	141.60	447,489.97	**1.0000**
2022	0.0535	**0.2915**	656,740.30	153.10	428,961.66	**0.9586**

(*) The correlation between the "Mortgage Interest Rate (Normalized)" and the "ABM Median House Price (Deflated and Normalized)" is -0.8235, indicating a strong negative correlation between mortgage interest rates and ABM house prices.

Table 14 - Correlation of Household Income, Inflation and ABM House Prices

	(*) Nominal Household Income		(*) Inflation		(*) ABM Median House Price	
Year	**ABM Input ($)**	**Normalized (%)**	**CPI**	**Normalized (%)**	**ABM Output ($)**	**Normalized (%)**
1975	11600.00	**0.1554**	29.00	**0.1894**	48,278.46	**0.0735**
1976	12440.00	**0.1666**	31.10	**0.2031**	56,925.24	**0.0867**
1977	13660.73	**0.1830**	33.60	**0.2195**	65,785.41	**0.1002**
1978	14907.15	**0.1997**	36.60	**0.2391**	72,019.15	**0.1097**
1979	16087.59	**0.2155**	40.00	**0.2613**	76,906.89	**0.1171**
1980	18145.99	**0.2431**	44.00	**0.2874**	80,033.95	**0.1219**
1981	19872.26	**0.2662**	49.50	**0.3233**	84,166.03	**0.1282**
1982	21358.91	**0.2861**	54.90	**0.3586**	86,285.54	**0.1314**
1983	21798.10	**0.2920**	58.10	**0.3795**	87,457.08	**0.1332**
1984	22957.23	**0.3075**	60.60	**0.3958**	97,648.76	**0.1487**
1985	23958.39	**0.3209**	63.00	**0.4115**	109,766.14	**0.1671**
1986	24803.50	**0.3323**	65.60	**0.4285**	116,609.06	**0.1776**
1987	25800.00	**0.3456**	68.50	**0.4474**	123,412.00	**0.1879**
1988	27492.55	**0.3683**	71.20	**0.4651**	138,396.07	**0.2107**
1989	29319.42	**0.3928**	74.80	**0.4886**	139,858.05	**0.2130**
1990	29471.53	**0.3948**	78.40	**0.5121**	144,131.49	**0.2195**
1991	29493.72	**0.3951**	82.80	**0.5408**	144,863.26	**0.2206**
1992	30227.74	**0.4049**	84.00	**0.5487**	161,429.68	**0.2458**
1993	29991.24	**0.4018**	85.60	**0.5591**	174,576.74	**0.2658**
1994	30213.94	**0.4047**	85.70	**0.5598**	167,779.57	**0.2555**
1995	30883.80	**0.4137**	87.60	**0.5722**	163,125.22	**0.2484**
1996	30822.99	**0.4129**	88.90	**0.5807**	181,636.76	**0.2766**
1997	31211.09	**0.4181**	90.40	**0.5905**	198,314.59	**0.3020**
1998	32388.18	**0.4339**	91.30	**0.5963**	213,777.92	**0.3255**
1999	34447.59	**0.4615**	92.90	**0.6068**	223,365.00	**0.3401**
2000	35583.50	**0.4767**	95.40	**0.6231**	221,457.71	**0.3372**
2001	37977.81	**0.5087**	97.80	**0.6388**	235,237.52	**0.3582**
2002	38759.12	**0.5192**	100.00	**0.6532**	259,289.58	**0.3948**
2003	39769.34	**0.5327**	102.80	**0.6715**	275,652.55	**0.4197**
2004	40810.07	**0.5467**	104.70	**0.6839**	299,093.49	**0.4554**
2005	42643.80	**0.5712**	107.00	**0.6989**	304,377.68	**0.4635**
2006	44436.35	**0.5953**	109.10	**0.7126**	307,125.35	**0.4677**
2007	46634.67	**0.6247**	111.50	**0.7283**	317,142.42	**0.4829**
2008	48388.39	**0.6482**	114.10	**0.7453**	315,267.85	**0.4800**
2009	48432.12	**0.6488**	114.40	**0.7472**	334,031.61	**0.5086**
2010	48895.99	**0.6550**	116.50	**0.7609**	389,026.13	**0.5924**
2011	50147.96	**0.6718**	119.90	**0.7831**	396,850.01	**0.6043**
2012	52233.28	**0.6997**	121.70	**0.7949**	404,821.43	**0.6164**
2013	53063.94	**0.7108**	122.80	**0.8021**	423,672.37	**0.6451**
2014	55380.44	**0.7419**	125.20	**0.8178**	475,563.90	**0.7241**
2015	55814.89	**0.7477**	126.60	**0.8269**	503,297.53	**0.7664**
2016	56608.47	**0.7583**	128.40	**0.8387**	563,765.94	**0.8584**
2017	58917.96	**0.7893**	130.40	**0.8517**	574,356.34	**0.8746**
2018	60468.18	**0.8100**	133.40	**0.8713**	536,037.40	**0.8162**
2019	61944.53	**0.8298**	136.00	**0.8883**	525,751.21	**0.8005**
2020	66800.00	**0.8948**	137.00	**0.8948**	547,153.72	**0.8331**
2021	69042.92	**0.9249**	141.60	**0.9249**	633,645.80	**0.9648**
2022	74650.22	**1.0000**	153.10	**1.0000**	656,740.30	**1.0000**

(*) The correlation between the "Nominal Household Income (Normalized)" and the "ABM Median House Price (Normalized)" is 0.9807, indicating a strong positive correlation between the household incomes and ABM house prices.
The correlation between the "Inflation (CPI, Normalized)" and the "ABM Median House Price (Normalized)" is 0.9376, indicating a strong positive correlation between the CPI and ABM house prices.

REFERENCES

Allen, J. J., Amano, R. A., Byrne, D. P., & Gregory, A. (2007). Canadian City Housing Prices and Urban Market Segmentation. *Bank of Canada Working Paper 2006-49*.

Bank for International Settlements. (2022, September 18). *Real Residential Property Prices for Canada [QCAR628BIS]*. Retrieved from FRED, Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/series/QCAR628BIS

Bank for International Settlements. (2022, September 18). *Residential Property Prices for Canada [QCAN628BIS]*. Retrieved from FRED, Federal Reserve Bank of St. Louis: https://fred.stlouisfed.org/series/QCAN628BIS

Bank of Canada. (2022, September 18). *Interest rates posted for selected products by the major chartered banks*. Retrieved from Banking and Financial Statistics: https://www.bankofcanada.ca/rates/banking-and-financial-statistics/posted-interest-rates-offered-by-chartered-banks/

Baptista, R., Farmer, J., Hinterschweiger, M., Low, K., Tang, D., & Uluc, A. (2016). Macroprudential policy in an agent-based model of the UK housing market. *Bank of England working papers No. 619*.

Canada Mortgage and Housing Corporation. (2018, June 22). *13.6% of Urban Households Were in Core Housing Need in 2016*. Retrieved from Housing Observer: https://www.cmhc-schl.gc.ca/en/housing-observer-online/2018-housing-observer/13-point-6-percent-urban-households-were-core-housing-need-2016

Canada Mortgage and Housing Corporation. (2022, October 22). *Calculating GDS / TDS*. Retrieved from https://www.cmhc-schl.gc.ca/en/professionals/project-funding-and-mortgage-financing/mortgage-loan-insurance/calculating-gds-tds

Canada Mortgage and Housing Corporation. (2022, September 18). *Housing markets to moderate from historic 2021 levels*. Retrieved from CMHC Media Newsroom: https://www.cmhc-schl.gc.ca/en/media-newsroom/news-releases/2022/housing-markets-moderate-historic-2021-levels

Carstensen, C. L. (2015, September 2). *An agent-based model of the housing market Steps toward a computational tool for policy analysis.* Retrieved from Analysis and Policy Observatory: https://apo.org.au/node/173221

Cheung, C. (2014). Deconstructing Canada's Housing Markets: Finance, Affordability and Urban Sprawl. *OECD Economics Department Working Papers, No. 1145*.

Crawford, A., Meh, C., & Jie, Z. (2013). The Residential Mortgage Market in Canada: A Primer. *Bank of Canada, Financial System Review* , 53-63.

Crowe, C., Giovanni, D., Igan, D., & Rabanal, P. (2013). How to deal with real estate booms: Lessons from country experiences. *Journal of Financial Stability 9*, 300-319.

De Grauwe, P. (2010). Top-down versus Bottom-up Macroeconomics. *CESifo Economics Studies, Vol. 56*, pp. 465-497.

Eviews. (2022). *Eviews 12 - Student Version Lite*. Retrieved from https://www.eviews.com/

Ge, J. (2013). Endogenous Rise and Collapse of Housing Prices. *ISU General Staff Papers, Iowa State University, Department of Economics*.

Ge, J. (2017). Endogenous rise and collapse of housing price An agent-based model of the housing market. *Computers Environment and Urban Systems 62*, 182-198.

Geanakoplos, John D.; Axtell, Robert; Farmer, J. Doyne; Howitt, Peter; Conlee, Benjamin; Goldstein, Jonathan; Hendrey, Matthew; Palmer, Nathan; Yang, Chun-Yi. (2012). Getting at Systemic Risk Via an Agent-Based Model of the Housing Market. *American Economic Review: Papers & Proceedings*, 53–58.

Gellatly, G., & Richards, E. (2019, March 26). Indebtedness and Wealth Among Canadian Households. *Economic Insights, no. 089, Statistics Canada*.

Gilbert, N., Hawksworth, J. C., & Swinney, P. (2008). *An agent-based model of the UK housing market*. Retrieved from University of Surrey: http://cress.soc.surrey.ac.uk/housingmarket/index.html

Gilbert, N., Hawksworth, J. C., & Swinney, P. (2009). An Agent-Based Model of the English Housing Market. *Association for the Advancement of Artificial Intelligence (www.aaai.org)*, 30-35.

Government of Canada. (2019, March 19). *An Affordable Place to Call Home*. Retrieved from Budget 2019: https://www.budget.gc.ca/2019/docs/themes/housing-logement-en.html

Government of Canada. (2021, April 19). *A Recovery Plan for Jobs, Growth, and Resilience*. Retrieved from Budget 2021: https://www.budget.gc.ca/2021/home-accueil-en.html

Government of Canada. (2022, April 7). *A Plan to Grow Our Economy and Make Life More Affordable*. Retrieved from Budget 2022: https://budget.gc.ca/2022/home-accueil-en.html

Gräbner, C. (2016). Agent-based computational models – a formal heuristic for institutionalist pattern modelling? *Journal of Institutional Economics 12*, 241-261.

Haldane, A. G., & Turrell, A. E. (2019). Drawing on different disciplines: macroeconomic agent-based models. *J Evol Econ*, pp. 39-66.

Hamill, L., & Gilbert, N. (2016). *Agent-Based Modelling in Economics.* John Wiley & Sons.

Himmelberg, C., Mayer, C., & Sinai, T. (2005). Assessing High House Prices: Bubbles, Fundamentals and Misperceptions. *Journal of Economic Perspectives, 19*, pp. 67-92.

Holland, J. H., & Miller, J. H. (1991). Artificial Adaptive Agents in Economic Theory. *The American Economic Review 81*, 365-370.

Lin, P.-t., & Fuerst, F. (2014). Volatility Clustering, Risk-Return Relationship, and Asymmetric Adjustment in the Canadian Housing Market. *Journal of Real Estate Portfolio Management* , pp. 37-46.

Lucas, R. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 19-46.

Macal, C. M., & North, M. J. (2010). Tutorial on agent-based modelling and simulation. *Journal of Simulation 4*, 151-162.

OSFI. (2017, October 17). *OSFI is reinforcing a strong and prudent regulatory regime for residential mortgage underwriting*. Retrieved from Office of the Superintendent of Financial Institutions (OSFI): http://www.osfi-bsif.gc.ca/Eng/osfi-bsif/med/Pages/B20_dft_nr.aspx

Rosenfield, A., Chingcuanco, F., & Miller, E. J. (2013). Agent-based Housing Market Microsimulation for Integrated Land Use, Transportation, Environment Model System. *Procedia Computer Science 19*, 841-846.

Siddall, E. (2019, March 5). *Are current mortgage rules too strict? No*. Retrieved from The Star: https://www.thestar.com/opinion/contributors/thebigdebate/2019/03/05/are-current-mortgage-rules-too-strict-no.html

Statistics Canada. (2018, December 14). *National balance sheet and financial flow accounts, third quarter 2018*. Retrieved from Statistics Canada: https://www150.statcan.gc.ca/n1/daily-quotidien/181214/dq181214a-eng.htm

Statistics Canada. (2019, March 5). *Spotlight on Canadians and Debt: Who's Vulnerable?* Retrieved from Statistics Canada: https://www150.statcan.gc.ca/n1/pub/11-627-m/11-627-m2019016-eng.htm

Statistics Canada. (2020, October 15). *Canadian Income Survey, 2020 (Released: 2022-03-23)*. Retrieved from https://www150.statcan.gc.ca/n1/daily-quotidien/220323/dq220323a-eng.htm

Statistics Canada. (2022, September 18). *Table 11-10-0190-01 Market income, government transfers, total income, income tax and after-tax income by economic family type*. Retrieved from https://www150.statcan.gc.ca/t1/tbl1/en/cv.action?pid=1110019001

Statistics Canada. (2022, September 18). *Table 18-10-0004-01 Consumer Price Index, monthly, not seasonally adjusted*. Retrieved from https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810000401

Statistics Canada. (2022, September 18). *Table 18-10-0005-01 Consumer Price Index, annual average, not seasonally adjusted*. Retrieved from https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810000501

Tesfatsion, L. (2002, March 15). *Agent-Based Computational Economics: Growing Economies from the Bottom Up*. Retrieved from ISU Economics Working Paper No. 1: http://www.econ.iastate.edu/tesfatsi/

Thaler, R. H. (2015). *Misbehaving: The making of behavioral economics.* New York: W W Norton & Co.

Turrell, A. (2016). Agent-Based Models: Understanding the Economy from the Bottom Up. *Bank of England Quarterly Bulletin 2016 Q4*, 173-188.

Waddell, P. (2002). UrbanSim: Modeling Urban Development for Land Use, Transportation and Environmental Planning. *Journal of the American Planning Association 68*, pp. 297-314.

Wakerly, E. C., Scott, B. G., & Nason, J. M. (2006). Common trends and common cycles in Canada: who knew so much has been going on? *Canadian Journal of Economics/Revue canadienne d'économique 39,* 320-347.

Wilensky, U. (1999). *NetLogo*. Retrieved from https://ccl.northwestern.edu/netlogo/